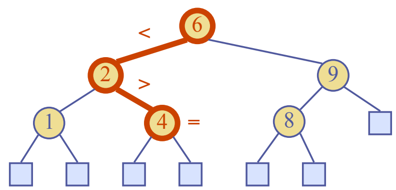
Binary Search Trees

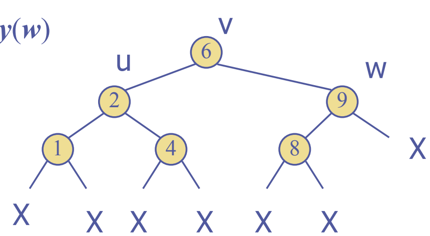


Fast Operations

- What if we could do search, insert, and remove in O(log n)?
  - Log 1,048,576 = 20

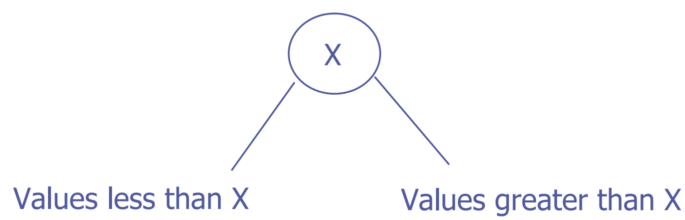
Binary Search Tree (§9.1)

- A binary search tree is a binary tree storing keys (or key-element pairs) satisfying the following property:
  - Let u, v, and w be three nodes such that u is in the left subtree of v and w is in the right subtree of v. We have
    $key(u) \leq key(v) \leq key(w)$



Binary Search Tree (§9.1)

- Property - given a node with a value X, all the values of nodes in the left subtree are smaller than X and all the values of the nodes in the right subtree are larger than X



Binary Search Tree (§9.1)

- External nodes do not store items? (Book says this)
  - Not a good way to look at the tree
    - Incredible waste of space
  - External nodes store items and children point to NULL
- An inorder traversal of a binary search trees visits the keys in increasing order

BST Operations

- makeFromEmpty - initialize a new tree
- isEmpty - return true if empty, false if not
- search - return pointer to node in which key is found, otherwise return NULL
- findMin - return smallest node value
- findMax - return largest node value

BST Operations

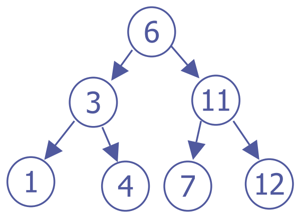
- insert - insert a new node into the tree maintaining BST property. All inserts are done at a leaf
- remove - remove a node from the tree maintaining BST property.
- display - print a tree in an order traversal

Array Implementation of a BST

- A BST can be implemented with an array
- Given a node i
  - parent(i) = (i - 1)/2
    - If i = 0, then no parent since root
  - leftChild(i) = 2i+1
    - If 2i+1 <= N, otherwise no child
  - rightChild(i) = 2i+2
    - If 2i+2 <= N, otherwise no child

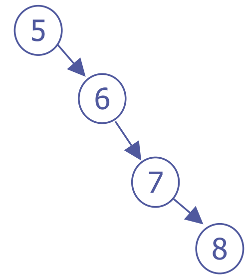
Array Implementation of a BST

0	1	2	3	4	5	6	7
6	3	11	1	4	7	12	



Array Implementation of a BST

- In class exercise - show the array for the following tree



Linked Implementation of a BST

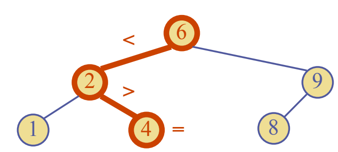
- Array disadvantages
  - Wasted space
  - Not enough space
- Linked implementation
  - Similar to linked list - size can grow and shrink easily during runtime

```
class Node {
friend class Tree;
private:
        itemtype item
        Node* left
        Node* right
        Node* parent
};
```

```
class Tree {
private:
        Node* root
        // internal functions
public:
        // functions for
        // interface
};
```


Search (§9.1.1)

- To search for a key *k*, we trace a downward path starting at the root
- The next node visited depends on the outcome of the comparison of *k* with the key of the current node
- If we reach a leaf, the key is not found and we return null
- Example: find(4)



Search

Recursive implementation of search

```
Node* search ( Node* nodePtr, itemtype key )
        if (nodePtr == NULL)
                return NULL
        else if ( nodePtr->item == key )
                return nodePtr
        else if ( nodePtr->item > key )
                return search(nodePtr->left, key)
        else
                return search(nodePtr->right, key)
```

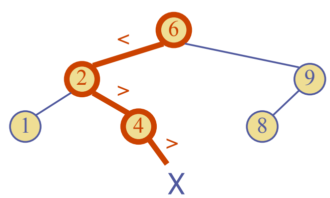

Inorder Traversal

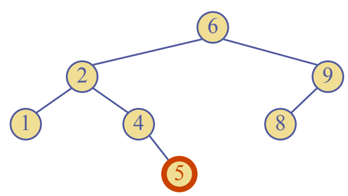
Recursive implementation of inorder traversal

```
void inorder(node* nodePtr)
        if ( nodePtr != NULL )
                inorder (nodePtr->left)
                print node
                inorder (nodePtr->right)
```

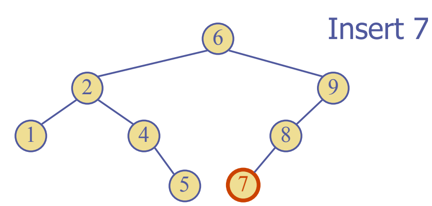

Insertion (§9.1.2)

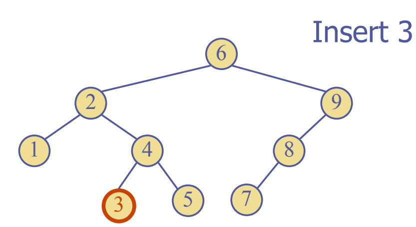
- To perform operation insertItem(k, o), we search for the position k would be in if it were in the tree
- All insertions create a new leaf node
- Example: insert 5





Insertion





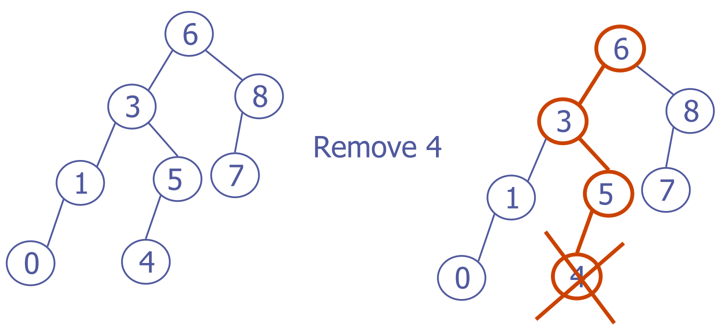
Insertion

- In class exercise - create a BST by inserted the following integers in the given order
  - 6 8 3 5 1 0 7 4

Deletion

- Traverse tree and search for node to remove
  - Five possible situations
    - Item not found
    - Removing a leaf
    - Removing a node with one child - right only
    - Removing a node with one child - left only
    - Removing a node with two children

Deletion - Removing a leaf



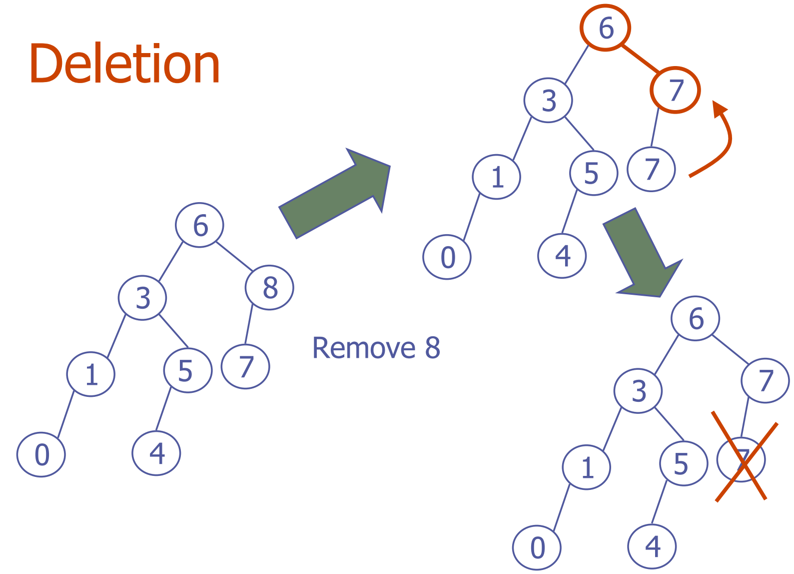
Deletion - Removing a node with children

- Otherwise the node has children - find replacement node
  - If the left child exists
    - Replace node information with the ***largest*** value smaller than the value to remove
      - findMax(leftChild)
  - Else there is a right child
    - Replace node information with the ***smallest*** value larger than value to remove
      - findMin(rightChild)

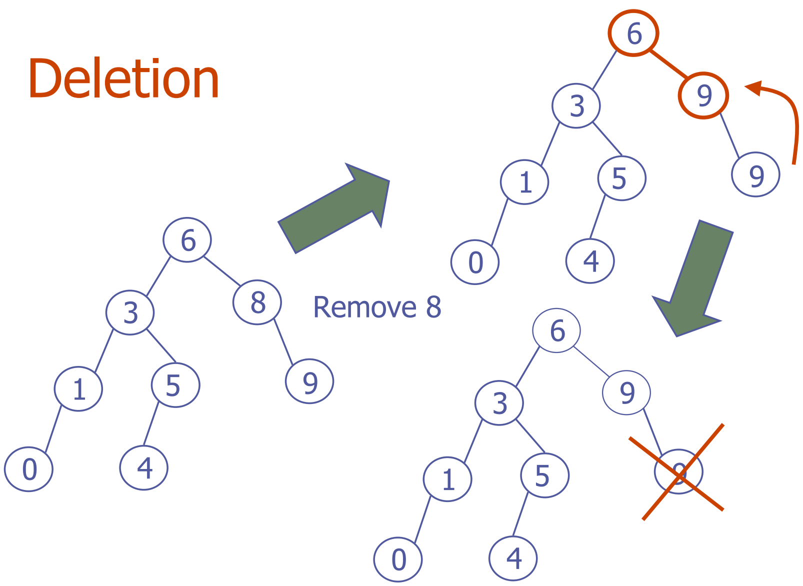
Deletion - Removing a node with children (continued)

- Splice out replacement node (call remove recursively)
- Just copy in info of replacement node over the value to remove (overload = if necessary)
  - Note - this is NOT the best solution if you have a large data structure. The overhead of the copy is too great and you should move the node instead.
- Delete replacement node if leaf

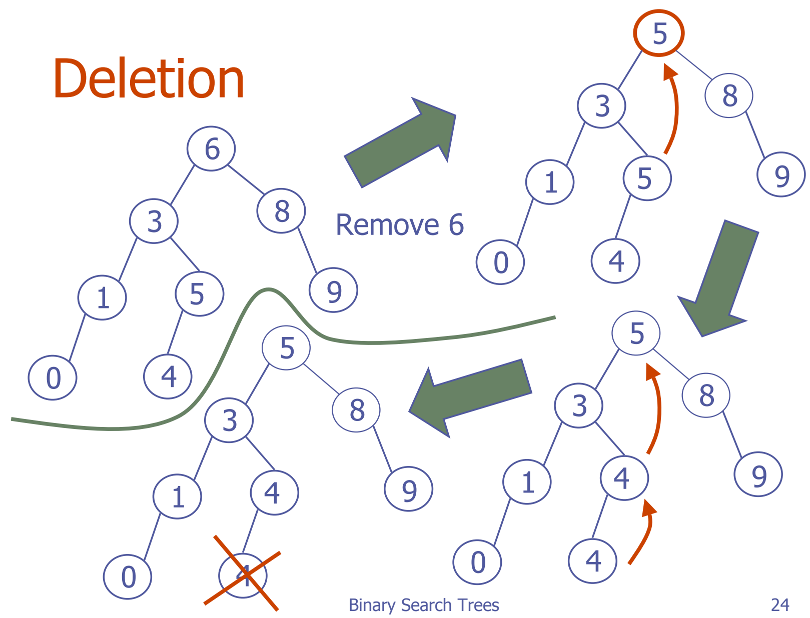
Deletion



Deletion



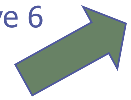
Deletion

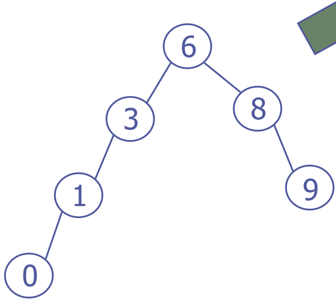


Deletion

In class exercise

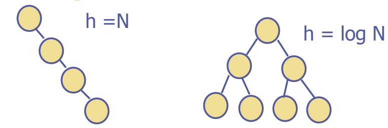
Remove 6





Analysis of BST Operations

◆In class exercise



	Worst Case	Average Case
empty		
search		
findMin		
findMax		
insert		
remove		
display		
makeFromEmpty		

Analysis of BST Operations

- Given a random ordering of insertions and deletions, the height of the tree will be quite close to log n
- We will learn later how to ensure the average case running times are also the worst case running times

Treesort

- Uses a BST to sort records efficiently
  - Use makeFromEmpty
    - Read in elements and insert in that order into a BST
  - Traverse inorder to read out nodes in ascending order
- Runtime
  - Average case - O(N log N)
  - Worst case - $O(N^2)$