# Tópicos de Computação Gráfica

# Topics in Computer Graphics

**10509: Doutoramento em Engenharia Informática**

## Chap. 2 — Rasterization

Rasterization

# Outline

…:

- Raster display technology.
- Basic concepts: pixel, resolution, aspect ratio, dynamic range, image domain, object domain.
- Rasterization and direct illumination.
- Graphics primitives and OpenGL.
- Geometry representations: explicit, parametric and implicit forms.
- Rasterization algorithms for straight line segments, circles and ellipses.
- Rasterization algorithms for triangles and polygons.
- Rasterization versus shading.

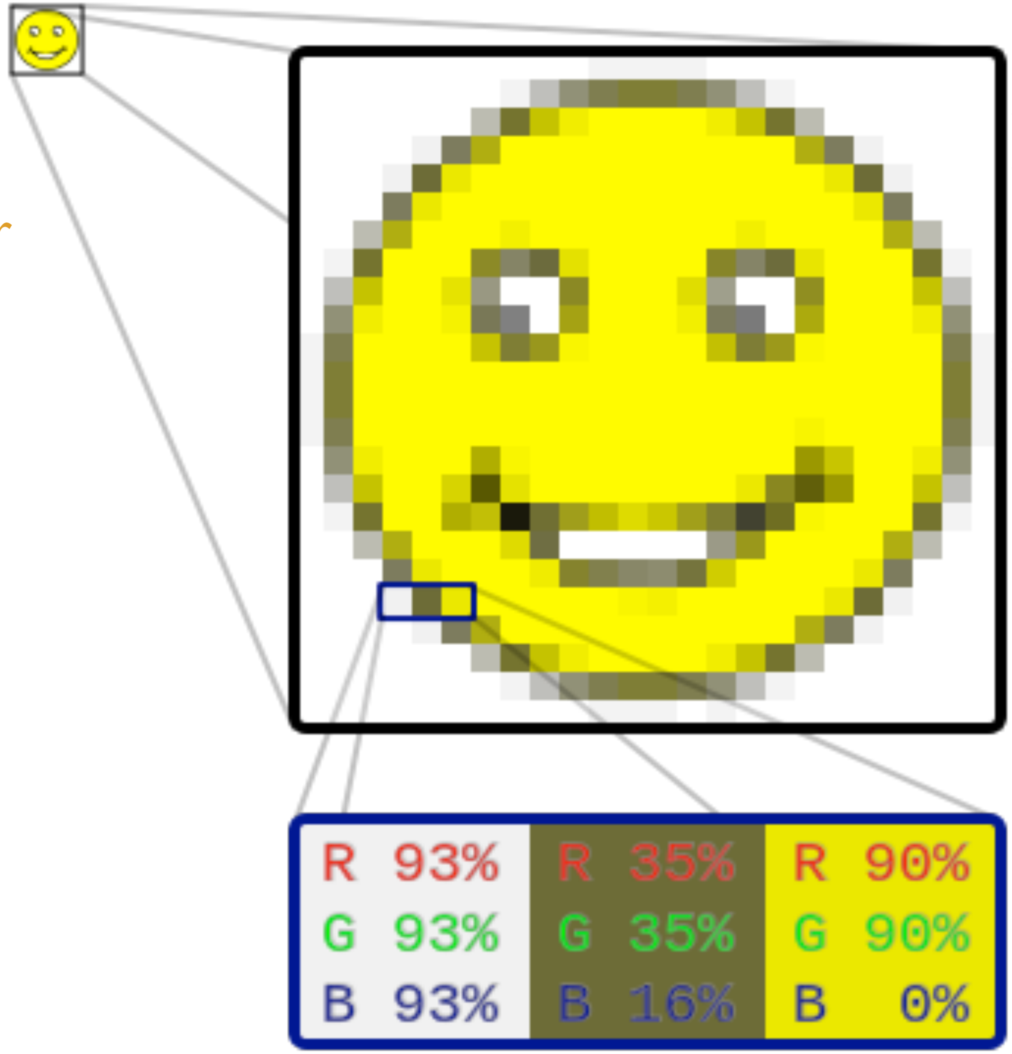
# Raster display

**Definition:**

- Discrete grid of elements (frame buffer of pixels).
  - Shapes drawn by setting the “right” elements
  - Frame buffer is scanned, one line at a time, to refresh the image (as opposed to vector display)

**Properties:**

- Difficult to draw smooth lines
- Displays only a discrete approximation of any shape
- Refresh of entire frame buffer




http://en.wikipedia.org/wiki/Raster_graphics

# Terminology

**Pixel: Picture Element**

- Smallest accessible element in picture.
- Usually rectangular or circular.

**Aspect Ratio:**

- Ratio between physical dimensions of pixel (not necessarily 1).

**Dynamic Range:**

- Ratio between minimal (not zero!) and maximal light intensity emitted by displayed pixel (black and white, respectively)

**Resolution:**

- Number of distinguishable rows and columns on a device measured in:
  - Absolute values (nxm)
  - Relative values (e.g., 300 dpi)
- Usually rectangular or circular.

**Screen space:**

- Discrete 2D Cartesian coordinate system of screen pixels.

**Object space:**

- Discrete 3D Cartesian coordinate system of the domain or scene or the objects live in.

# SCAN CONVERSION

# Scan conversion / rasterization (for *direct illumination*)

**Definition:**

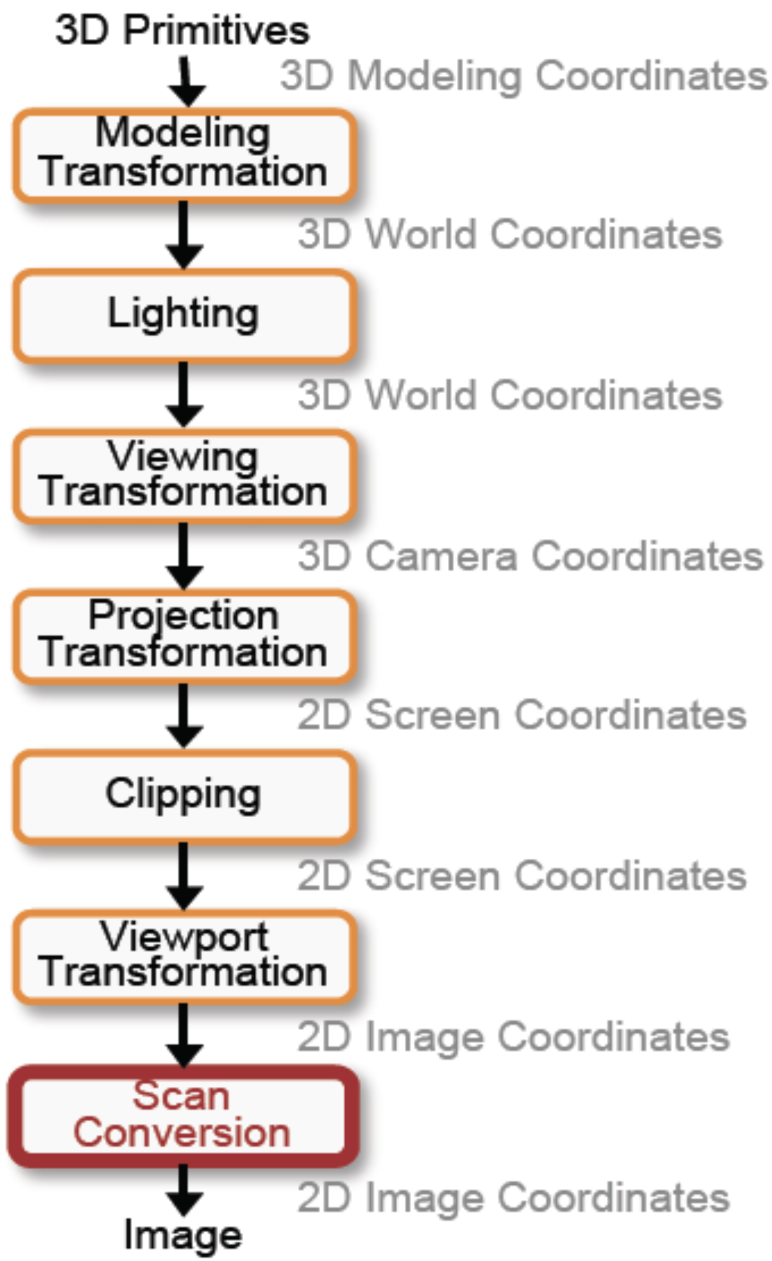
- The process of converting geometry into pixels.
- Final step in pipeline: *rasterization (scan conversion)*
- From screen coordinates (float) to pixels (int)
- Writing pixels into frame buffer.

**Scan conversion:**

- Figuring out which pixels to turn on.

**Shading:**

- Determine a color for each filled pixel.



# Graphics primitives

**OpenGL Primitive Taxonomy:**

- Point: POINTS
- Line: LINES, LINE_STRIP, LINE_LOOP
- Triangle: TRIANGLES, TRIANGLE_STRIP, TRIANGLE_FAN
- Polygon: QUADS, QUAD_STRIP, POLYGON

**Other Primitives:**

- Arc
- Circle
- Ellipsis
- Generic Curves

How is each geometric primitive really drawn on screen?

# Geometric representations for lines in $IR^2$

## Explicit form:

$$y = f(x) = mx + b$$

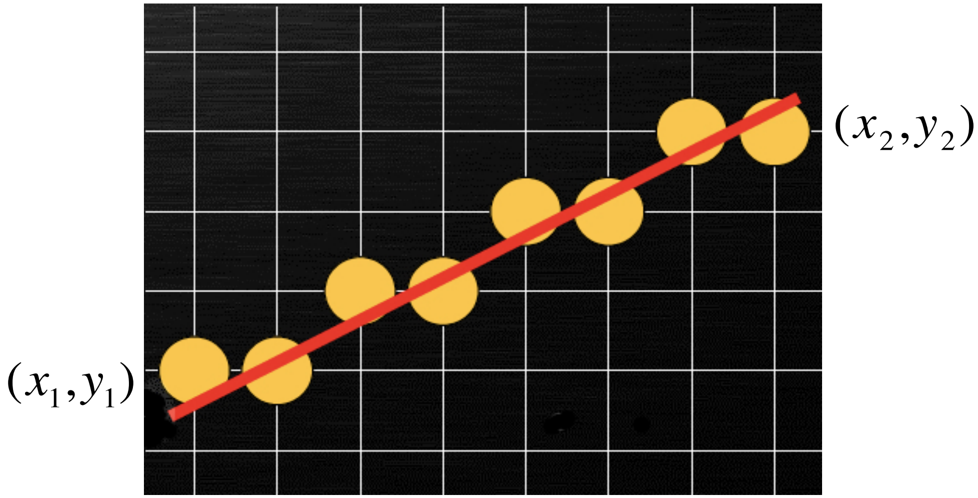
## Implicit form:

$$f(x,y) = Ax + By + C = 0$$

## Parametric form:

$$x = x(t) = m_0 t + b_0$$
$$y = y(t) = m_1 t + b_1$$

# Scan converting lines

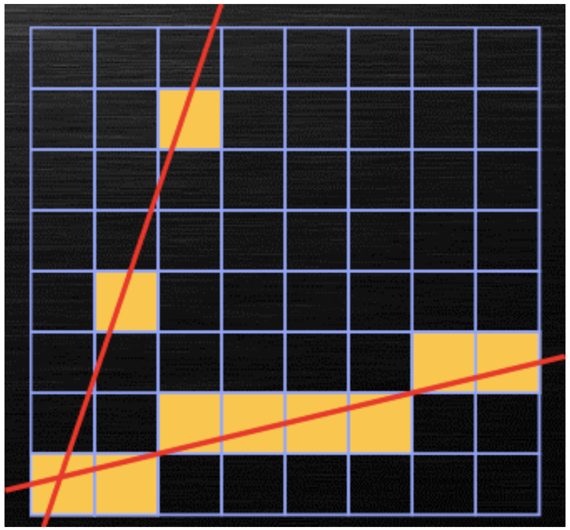



**Example:**

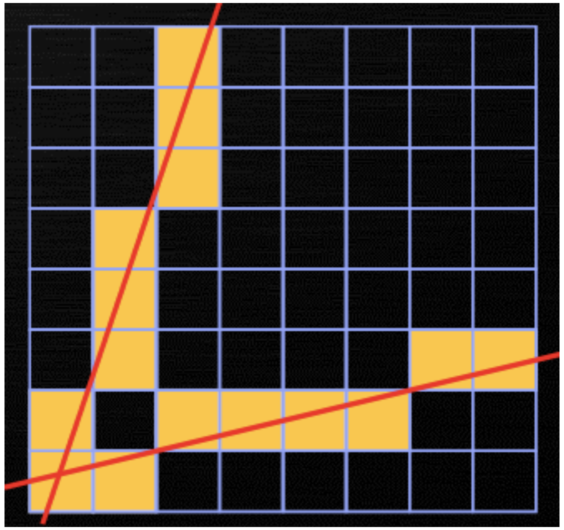
- Draw from $(x_1,y_1)$ to $(x_2,y_2)$

**Correctness/quality issues:**

- Gaps exist for line with slope m>1 (by varying x)



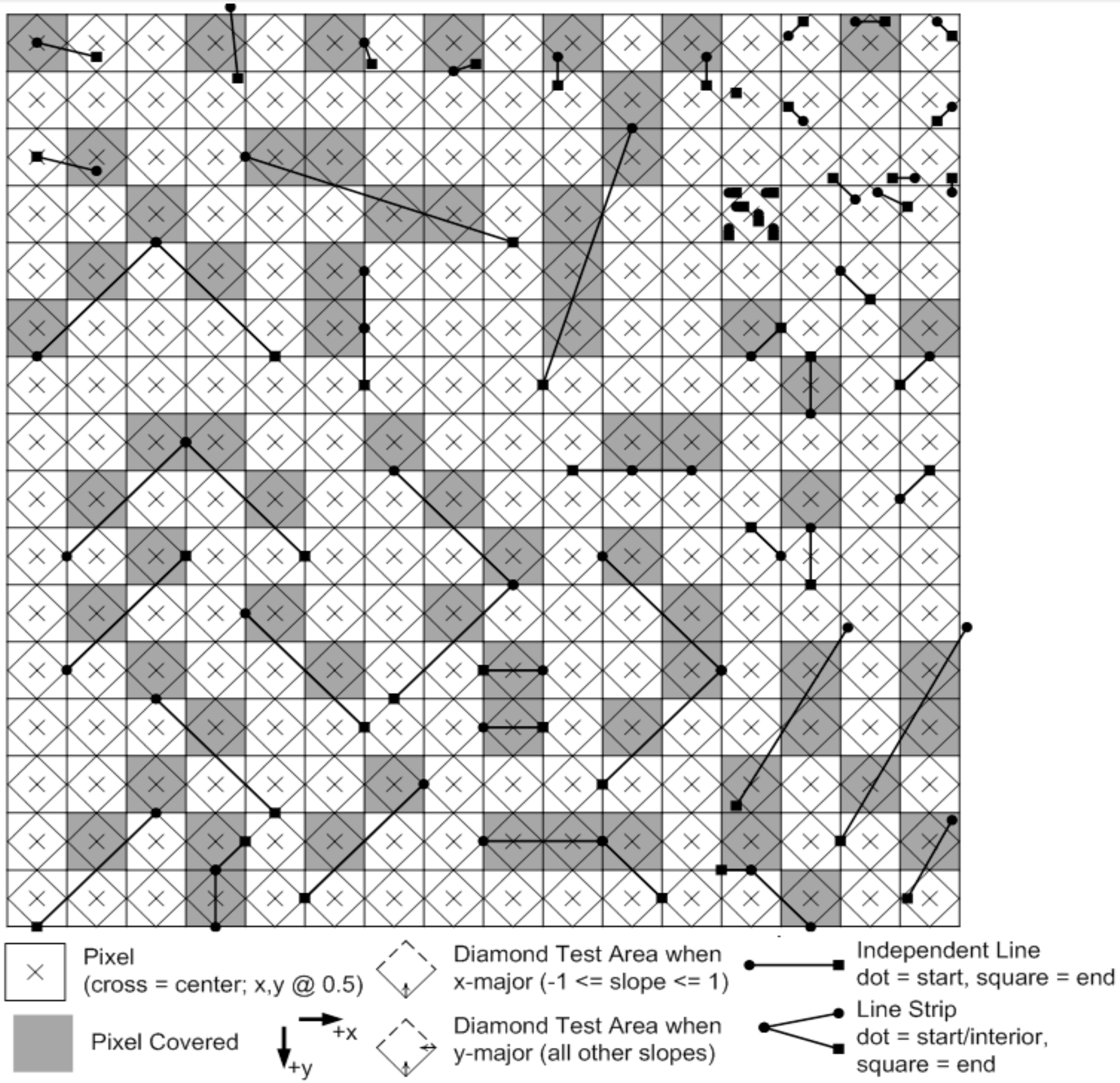
wrong (steeper line)



correct

# Rasterization rules

Line rasterization rules use a **diamond test area** to determine if a line covers a pixel.




https://msdn.microsoft.com/en-us/library/windows/desktop/cc627092%28v=vs.85%29.aspx#Line_1

# Direct scan conversion

**Explicit form:**

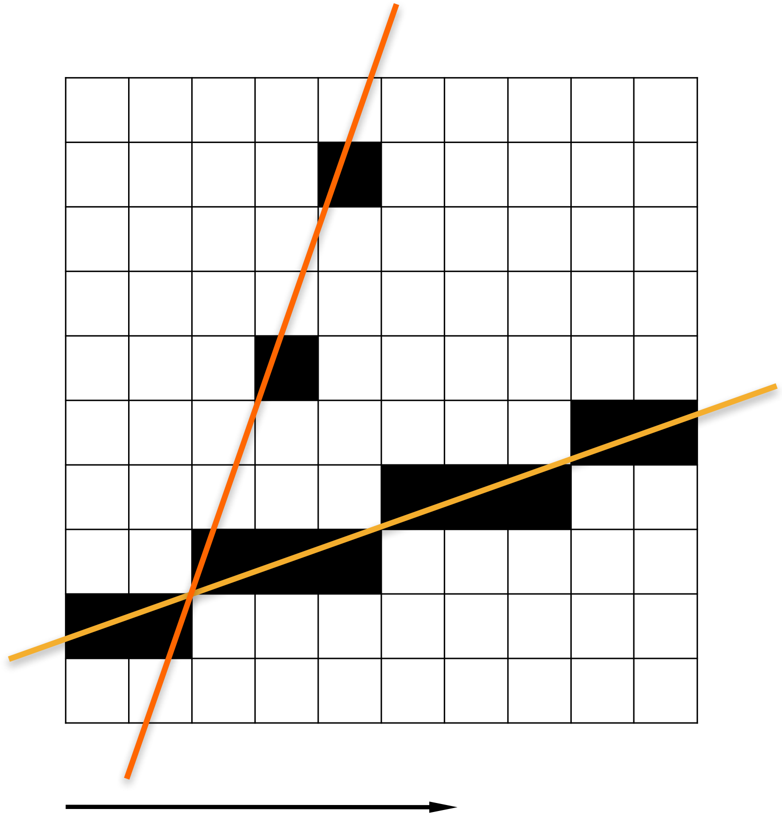
- $y=mx+b$, where $m=(y_{i+1}-y_i)/(x_{i+1}-x_i)=\Delta y/\Delta x$ and $0 \leq m \leq 1$ (1st, 4th, 5th and 8th octants)
- What else?

**Key idea:**

- Increment x from $x_i$ to $x_f$ and calculate the corresponding value $y=mx+b$

**Drawbacks:**

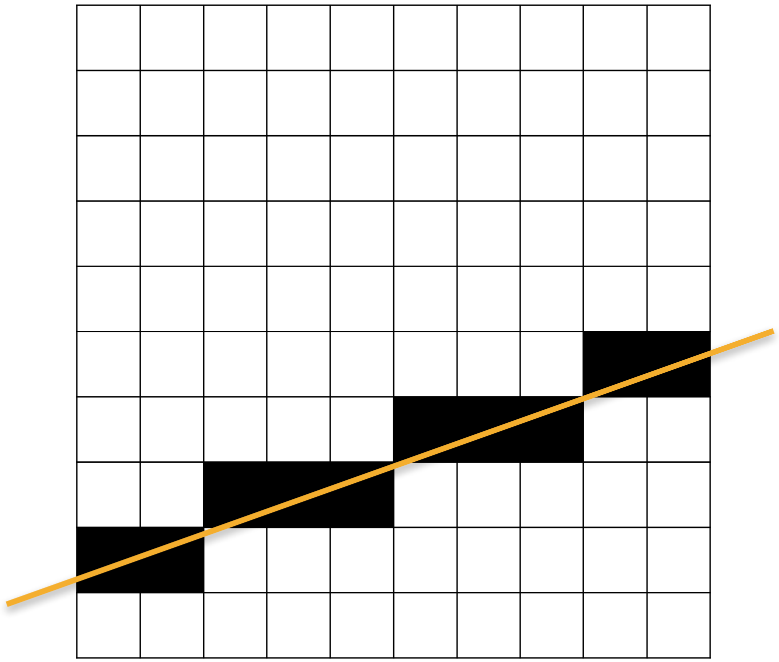
- Gaps when $m>1$. The solution is to increment y instead of x when $m>1$.
- Floating-point computations: floating-point multiplication and addition for every step in x.



# Direct scan conversion (cont'd)

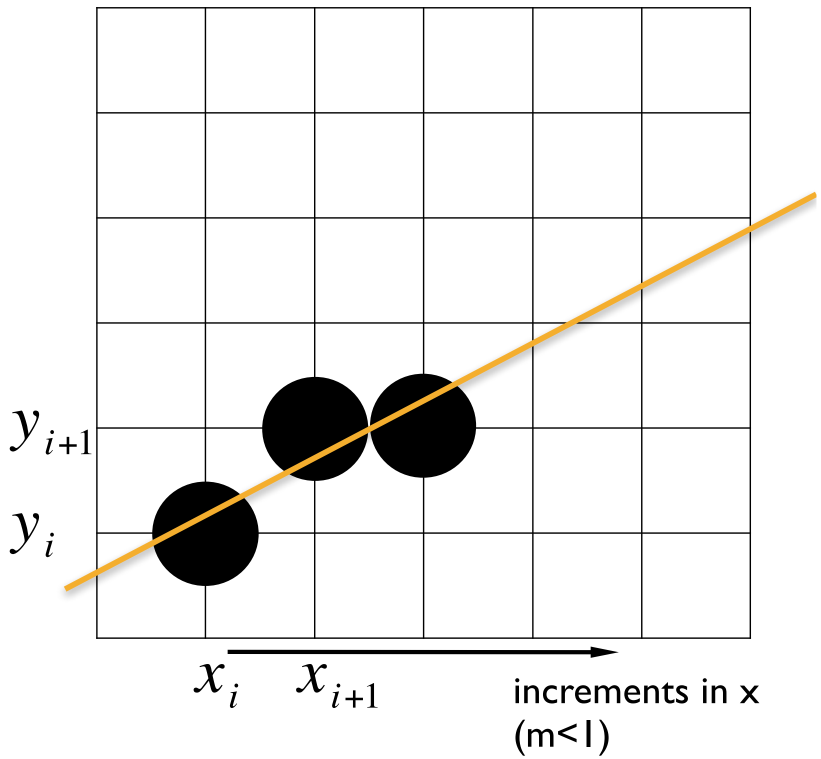
## Algorithm (m<1):

- $m=(y_f-y_i)/(x_f-x_i)$;
- $b=y_i-m*x_i$;
- x=xi; y=yi;
- DrawPixel(x,y);
- for ($x=x_i+1$; $x<=x_f$; x++).
  - y=m*x+b;
  - DrawPixel(x,y);



increments in x
(m<1)

# DDA algorithm (Digital Differential Analyser)




## Explicit form:

- $y=mx+b$, where $m=(y_{i+1}-y_i)/(x_{i+1}-x_i)=\Delta y/\Delta x$ and $0\leq m\leq 1$ (1st, 4th, 5th and 8th octants)

## Key idea:

- Increment x from $x_i$ to $x_f$ and calculate the corresponding value y:
- Current pixel: $y_i=mx_i+b$
- Next pixel:
  - $y_{i+1}=mx_{i+1}+b=m(x_i+1)+b=y_i+m$
  - Draw pixel $(x_{i+1},y_{i+1})$, where $y_{i+1}=\mathrm{ROUND}(y_{i+1})$

## Drawbacks:

- Gaps when m>1. In this case, increment y.
- Floating-point arithmetic: a floating-point addition and a round operation.

## Algorithm (m<1):

- $m=(y_f-y_i)/(x_f-x_i)$;
- x=xi; y=yi;
- DrawPixel(x,y);
- for $(x=x_i+1; x<=x_f; x++)$.
  - y=y+m;
  - DrawPixel(x,y);

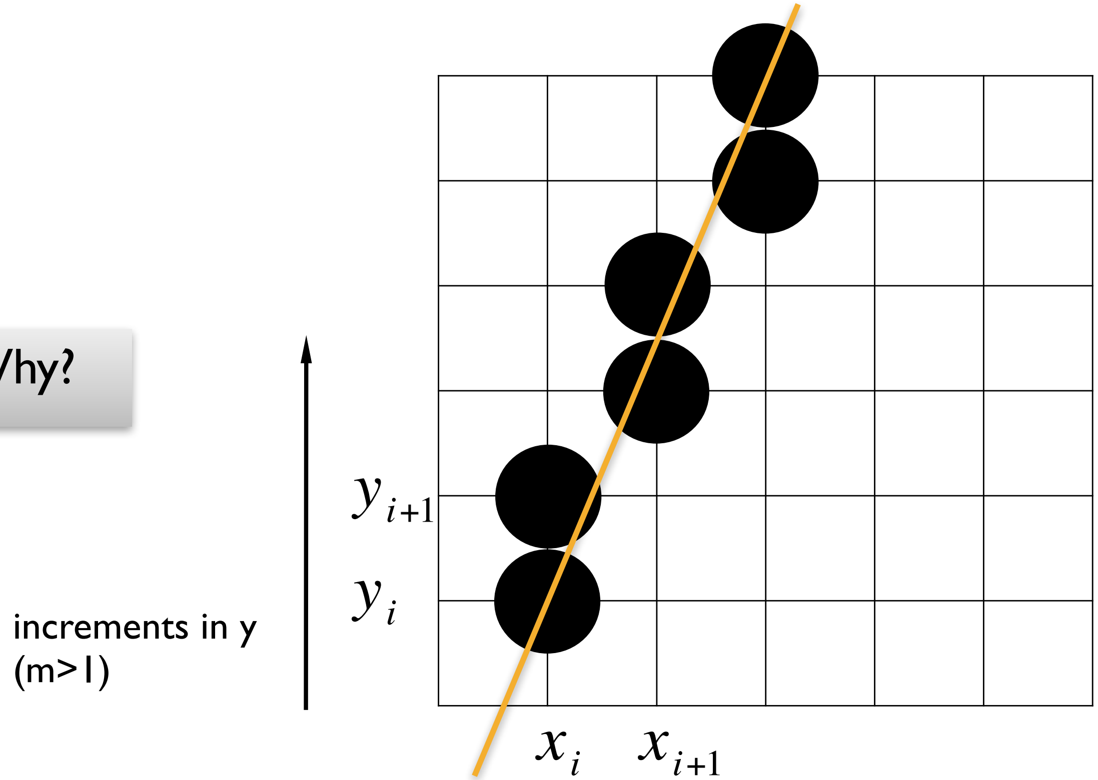
Note that the *explicit form* is not used directly!

# DDA algorithm (cont'd)

**Algorithm (m>1):**

- $m=(y_f-y_i)/(x_f-x_i)$;
- x=xi; y=yi;
- DrawPixel(x,y);
- for $(y=y_i+1;\ y<=y_f;\ y++)$.
  - x=x+**1/m**;
  - DrawPixel(x,y);

Why?




Note that the *explicit form* is not used directly!

# Bresenham algorithm

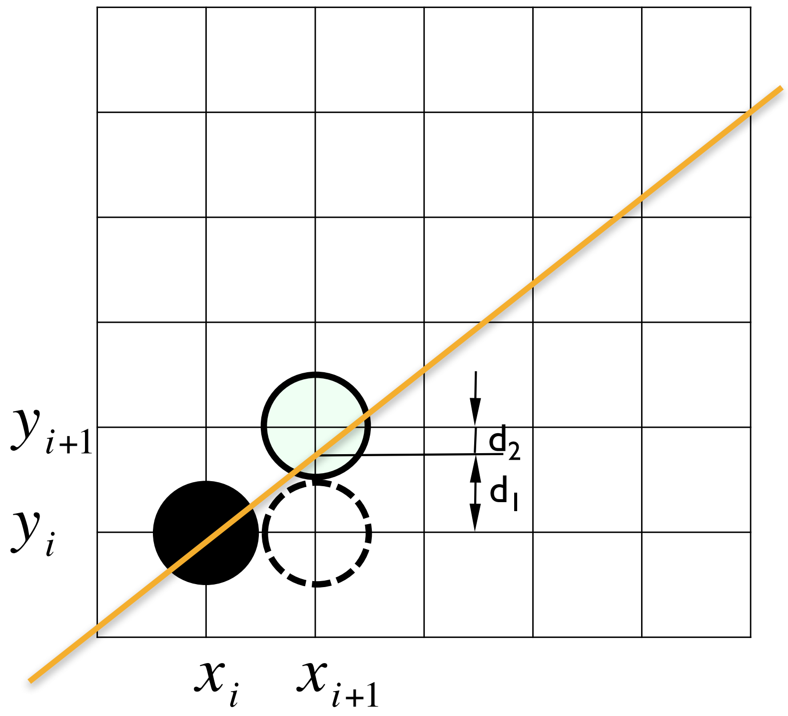
Bresenham, J.E. *Algorithm for computer control of a digital plotter, IBM Systems Journal, January 1965, pp. 25-30.*

## Explicit form:

- $y=mx+b$, where $m=\Delta y/\Delta x$ and $0 \leq m \leq 1$

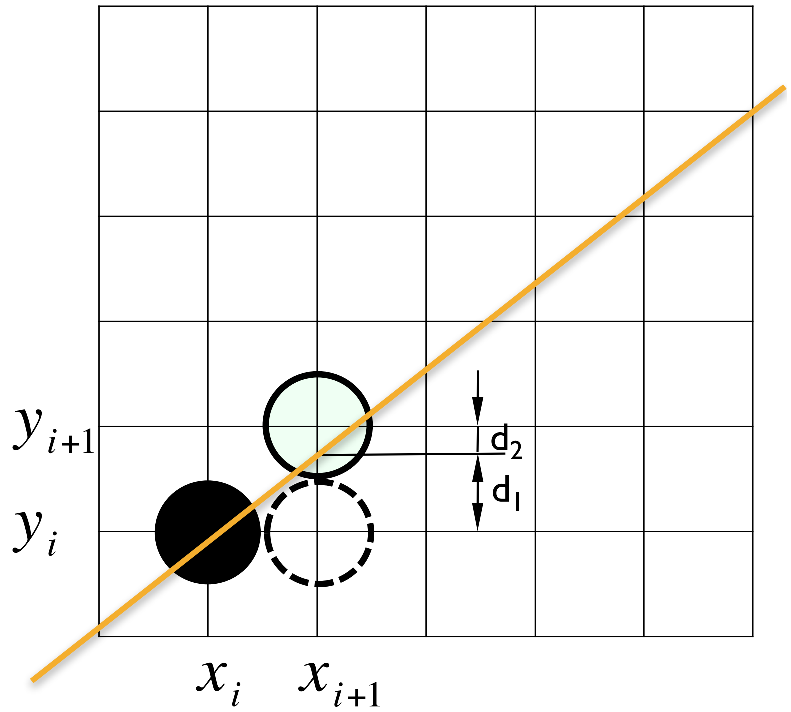
## Key idea:

- Increment x from $x_i$ to $x_f$ and calculate the corresponding value y.
- Current pixel: $(x_i,y_i)$
- Next pixel: either $(x_{i+1},y_i)$ or $(x_{i+1},y_{i+1})$
  - $d_1=y-y_i=mx_{i+1}+b-y_i=m(x_i+1)+b-y_i$
  - $d_2=y_{i+1}-y=y_i+1-y=y_i+1-m(x_i+1)+b$
  - $\Delta d=d_1-d_2=2m(x_i+1)-2y_i+2b-1$
  - If $\Delta d>0$ choose higher pixel $(x_{i+1},y_{i+1})$
  - If $\Delta d\leq 0$ choose lower pixel $(x_{i+1},y_i)$




Exact y-coordinate value $y=y_i+d_1$ of straight line at $x=x_{i+1}$

# Bresenham algorithm (cont'd)



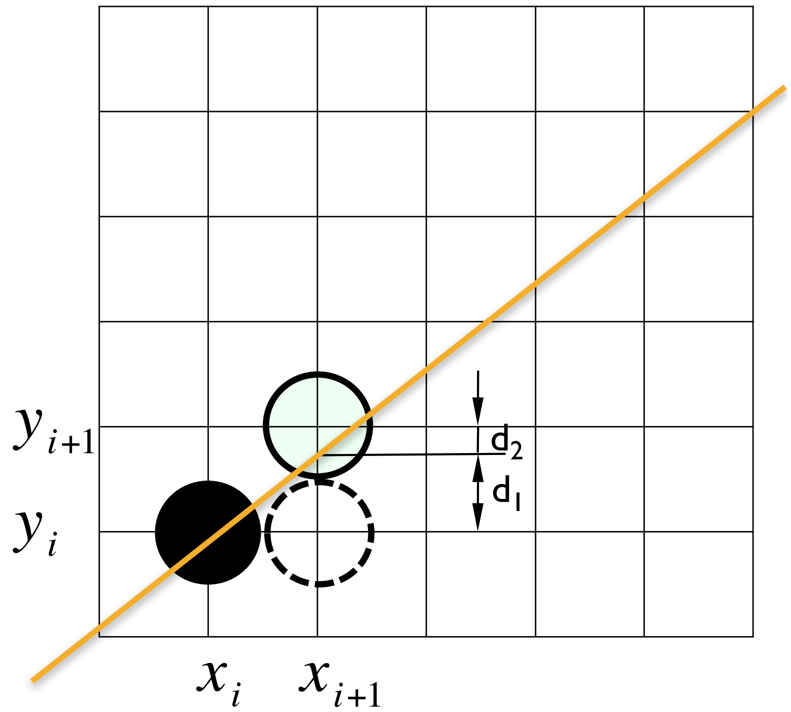

## Integer arithmetic (?):

- From triangle similarity, we know that
  - $m=\Delta y/\Delta x=d_1/(x_{i+1}-x_i)=d_1$
  - $d_2=1-d_1=1-m$
- Hence
  - $d_1-d_2=2m-1$
- To take advantage of <u>integer arithmetic</u>, we use the following decision parameter at the first pixel $(x_i,y_i)$ to choose which is the next pixel:
  - $p_i=\Delta x(d_1-d_2)=2\Delta y-\Delta x$
- But, in general terms, and using $d_1$ and $d_2$ in the previous page, we have:
  - $p_i=\Delta x(d_1-d_2)=2\Delta y.x_i-2\Delta x.y_i+K$, where K is a constant
- Consequently, the decision parameter at $(x_{i+1},y_{i+1})$ will be:
  - $p_{i+1}=2\Delta y.x_{i+1}-2\Delta x.y_{i+1}+K$ or
  - $p_{i+1}=p_i+2\Delta y(x_{i+1}-x_i)-2\Delta x(y_{i+1}-y_i)$ (note that $x_{i+1}-x_i=1$)

# Bresenham algorithm (cont'd)

## Algorithm:

```
void Bresenham (int xi, int yi, int xf, int yf)
{
	int x,y,dx,dy,p;
	x = xi; y = yi;
	p = 2 * dy - dx;
	for(x=xi; x<=xf; x++)
	{
		DrawPixel (x,y);
		if (p> 0)
		{
			y = y + 1;
			p = p - 2 * dx;
		}
		p= p + 2 * dy;
	}
}
```



# Midpoint algorithm

Bresenham, J.E. *Algorithm for computer control of a digital plotter, IBM Systems Journal, January 1965, pp. 25-30.*

**Implicit form:**

– $f(x,y)=Ax+By+C=0$

**Key idea:**

– Starting from $y=mx+b$, where $m=\Delta y/\Delta x$ and $0 \leq m \leq 1$, we have:

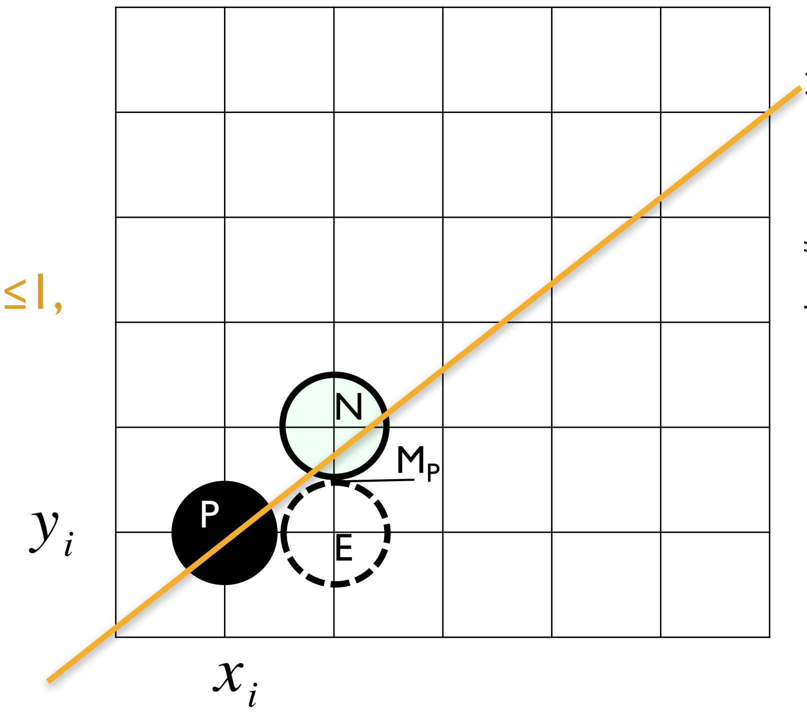
$$f(x,y)=\Delta y.x-\Delta x.y+b.\Delta x=0$$

with $A=\Delta y$, $B=-\Delta x$, and $C=b.\Delta x$

– Current pixel: $(x_i,y_i)$

– Next pixel: either $(x_{i+1},y_i)$ or $(x_{i+1},y_{i+1})$

  - Let the decision parameter $p_i=f(M_P)=f(x_i+1,y_i+1/2)$
  - If $p_i<0$ choose higher pixel $(x_{i+1},y_{i+1})$ at N
  - If $p_i \geq 0$ choose lower pixel $(x_{i+1},y_i)$ at E




Current pixel $P(x_i,y_i)$

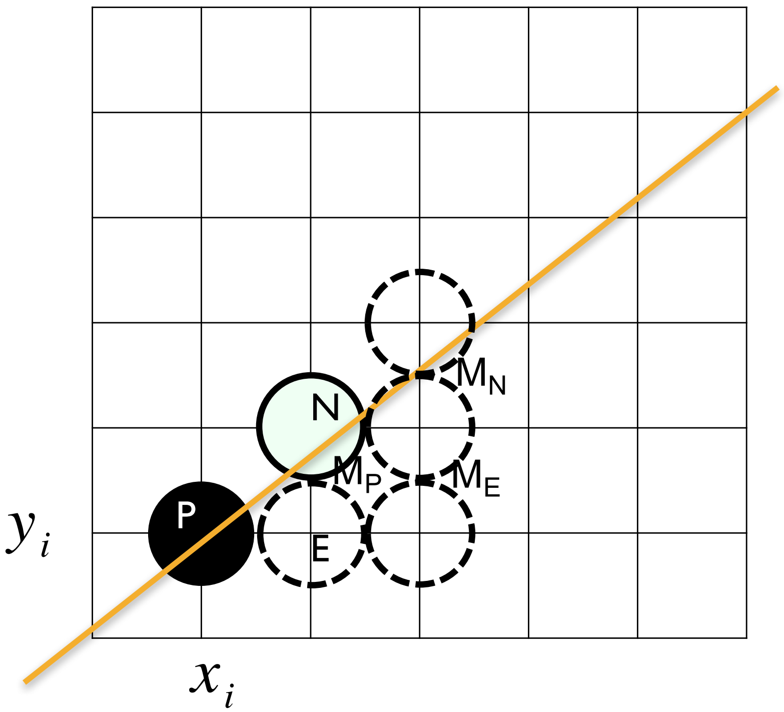
http://www.mat.univie.ac.at/~kriegl/Skripten/CG/node25.htm

# Midpoint algorithm (cont'd)

Let us now determine the relation between the function values at consecutive midpoints:

## Key idea (cont'd):

- $p_i=f(M_P)=f(x_i+1,y_i+1/2)=A.(x_i+1)+B.(y_i+1/2)+C$
- If E is chosen:
  - $p_{i+1}=f(M_E)=f(x_i+2,y_i+1/2)=A.(x_i+2)+B.(y_i+1/2)+C$
    $=p_i+A=p_i+\Delta y$
- If N is chosen:
  - $p_{i+1}=f(M_N)=f(x_i+2,y_i+3/2)=A.(x_i+2)+B.(y_i+3/2)+C$
    $=p_i+A+B=p_i+\Delta y-\Delta x$




Current pixel:
E (East) or
N (Nord-East)

## Integer arithmetic (?):

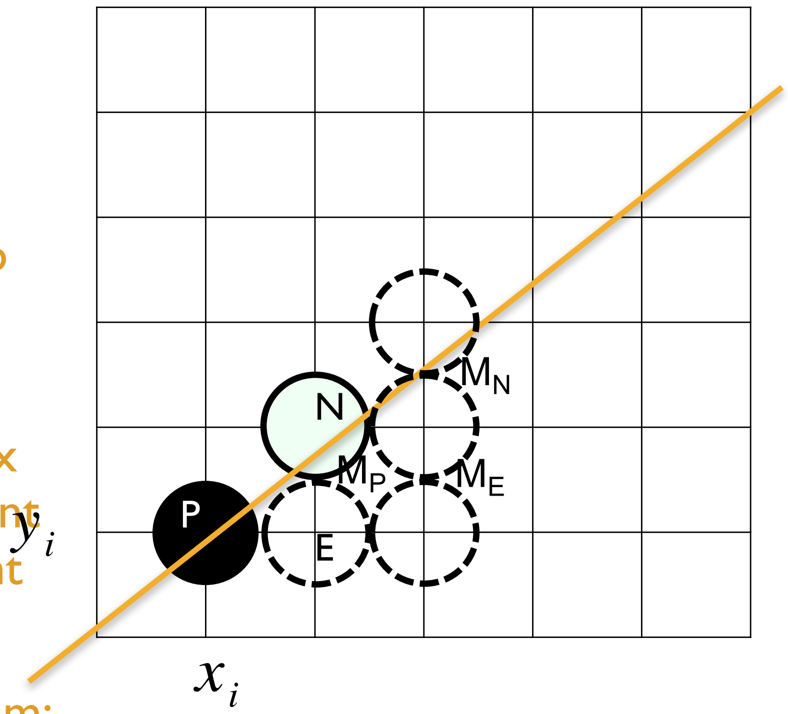
- Initial decision parameter:
  - $p_i=f(M_P)=f(x_i+1,y_i+1/2)=A.(x_i+1)+B.(y_i+1/2)+C$
    $=f(P)+A+B/2=f(P)+\Delta y-\Delta x/2=\Delta y-\Delta x/2$

*Multiplying the decision parameter by 2 we realize that we obtain exactly the Bresenham algorithm given before.*

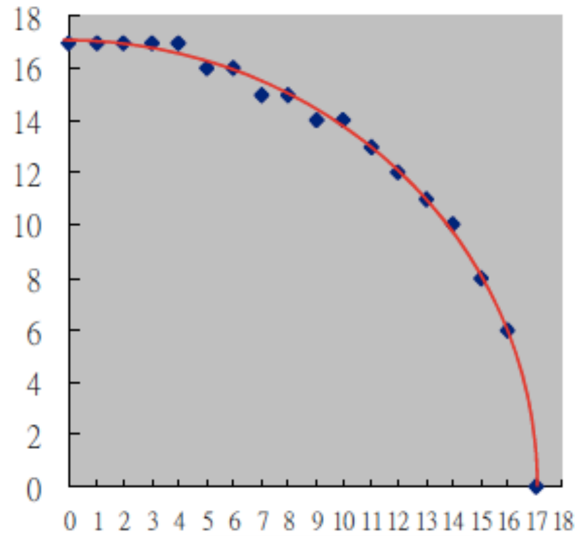
# General Bresenham's algorithm for lines

**To generalize lines with arbitrary slopes:**

- We need to consider symmetry between various octants and quadrants.
- For m>1, interchange roles of x and y, that is step in y direction, and decide whether the x value is above or below the line.
- If m>1, and right endpoint is the first point, both x and y decrease. To ensure uniqueness, independent of direction, always choose upper (or lower) point if the line go through the mid-point.
- Handle special cases without invoking the algorithm: horizontal, vertical and diagonal lines



# Scan converting circles




*gap problem*

**Explicit form:** $y = f(x) = \pm\sqrt{R^2 - x^2}$

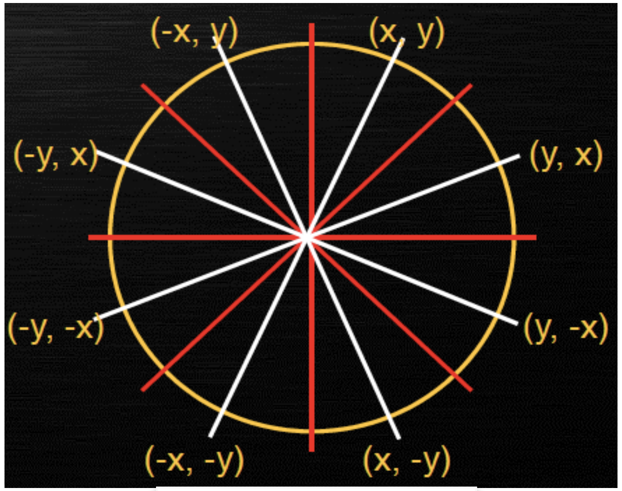
- Usually, we draw a quarter circle by incrementing x from 0 to R in unit steps and solving for +y for each step.

**Parametric form:** $\begin{cases} x = R\cos\theta \\ y = R\sin\theta \end{cases}$

- Done by stepping the angle from 0 to 90°.
- Solves the gap problem of explicit form.

**Implicit form:** $f(x,y) = x^2 + y^2 - R^2 = 0$

- If f(x,y)=0, then it is on the circle;
- If f(x,y)>0, then it is outside the circle;
- If f(x,y)<0, then it is inside the circle.




*8-way symmetry*

# Midpoint circle algorithm

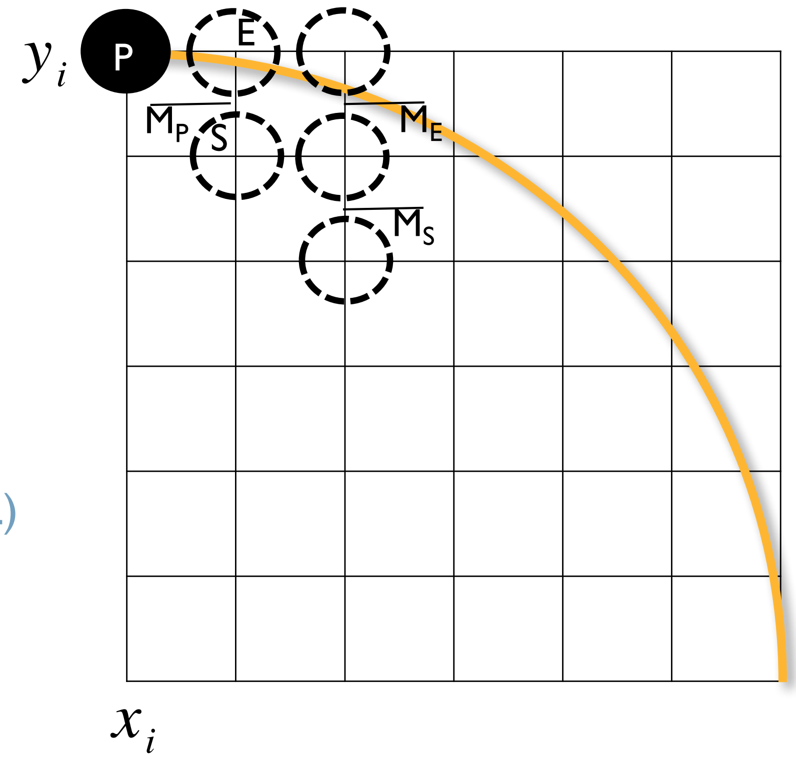
J.E. Bresenham. A linear algorithm for incremental digital display of circular arcs. *Communications of the ACM, 20(2):100-106, 1977.*

## Implicit form:

- $f(x,y)=x^2+y^2-R^2=0$

## Key idea:

- Current pixel: $P(x_i,y_i)$
- Next pixel: either $(x_{i+1},y_i)$ or $(x_{i+1},y_{i-1})$
  - Let the decision parameter $p_i=f(M_P)=f(x_i+1,y_i-1/2)$
  - If $p_i<0$ choose higher pixel $(x_{i+1},y_i)$ at E
  - If $p_i\geq 0$ choose lower pixel $(x_{i+1},y_{i-1})$ at S




Current pixel $P(x_i,y_i)$

http://www.mat.univie.ac.at/~kriegl/Skripten/CG/node25.htm

# Midpoint circle algorithm (cont'd)

Let us now determine the relation between the function values at consecutive midpoints:

## Key idea (cont'd):

– $p_i=f(M_P)=f(x_i+1,y_i-1/2)=(x_i+1)^2+(y_i-1/2)^2-R^2$

– If E is chosen:

- $p_{i+1}=f(M_E)=f(x_i+2,y_i-1/2)=(x_i+2)^2+(y_i-1/2)^2-R^2$
  $=p_i+(2x_i+3)$

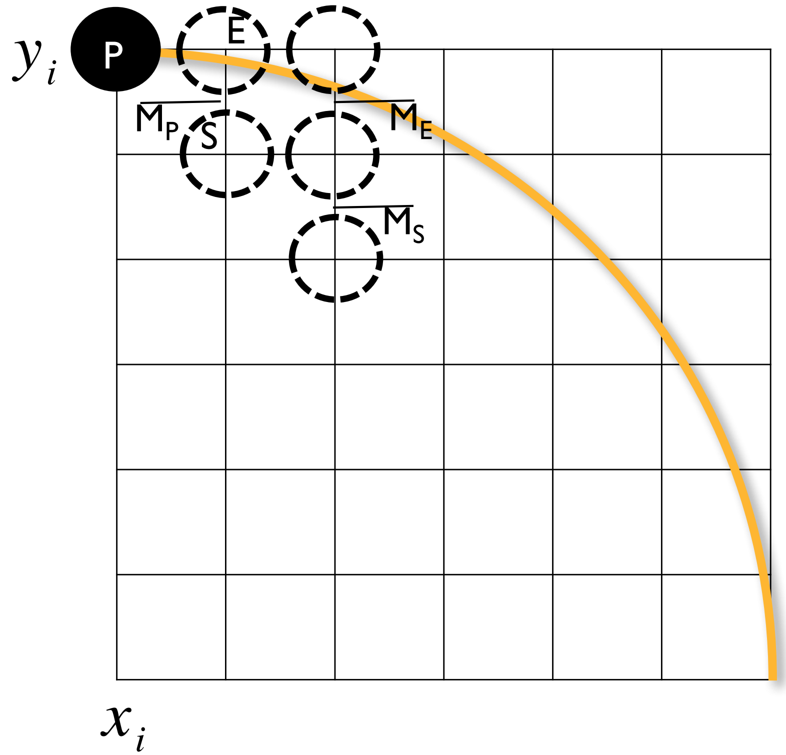
– If S is chosen:

- $p_{i+1}=f(M_S)=f(x_i+2,y_i-3/2)=(x_i+2)^2+(y_i-3/2)^2-R^2$
  $=p_i+(2x_i-2y_i+5)$

## Integer arithmetic:

– Initial decision parameter at $(x_i,y_i)=(0,R)$:

- $p_i=f(M_P)=f(x_i+1,y_i-1/2)=(x_i+1)^2+(y_i-1/2)^2-R^2$
  $=f(P)+2x_i-y_i+5/4=2x_i-y_i+5/4=5/4-R\cong 1-R$



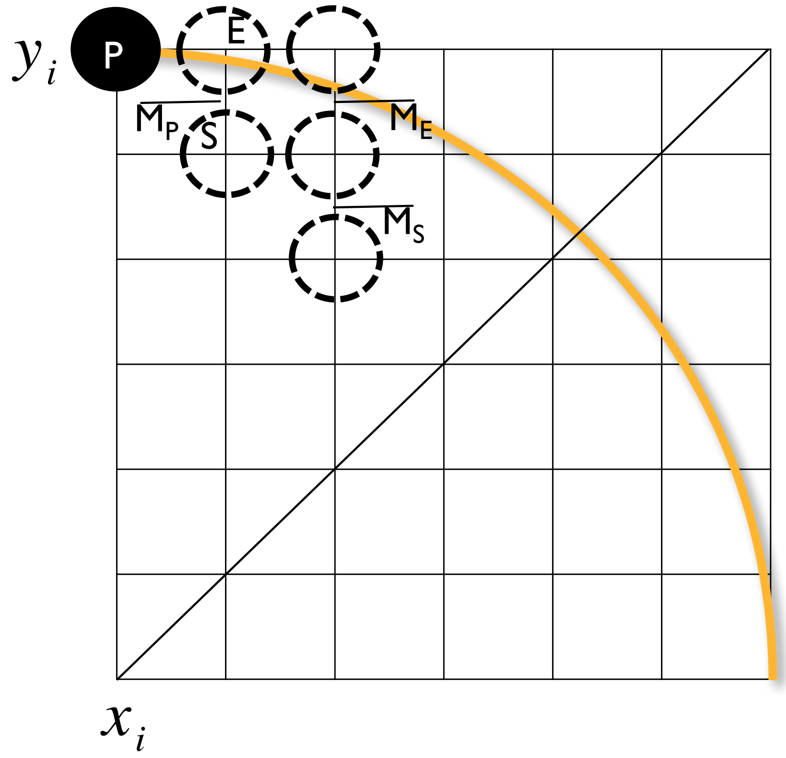

Current pixel:
E (East) or
S (South-East)

# Midpoint circle algorithm (cont'd)

## Algorithm:

```
void MidPointCircle(int R) {
        int x=0, y=R, d=1-R;

        DrawPixel(x,y);
        while (y>x)
        {
                if (p< 0)                       // select E
                        p=p + 2 * x + 3;
                else                            // select S
                {
                        p = p + 2 * (x - y) + 5;
                        y = y - 1;
                }
                x = x + 1;
                DrawPixel(x,y);
        }
}
```




The algorithm only calculates the pixels on the 2nd octant. The remaining pixels are found using 8-way-symmetry.

# SCAN CONVERSION

# of

# TRIANGLES/POLYGONS

# Scan converting of polygons

**Multiple tasks for scan conversion:**

- Filling polygon (inside/outside)
- Pixel shading (color interpolation)
- Blending (accumulation, not just writing)
- Depth values (z-buffer hidden-surface removal)
- Texture coordinate interpolation (texture mapping)

**Hardware efficiency critical**

**Many algorithms for filling (inside/outside)**

**Much fewer that handle all tasks well**

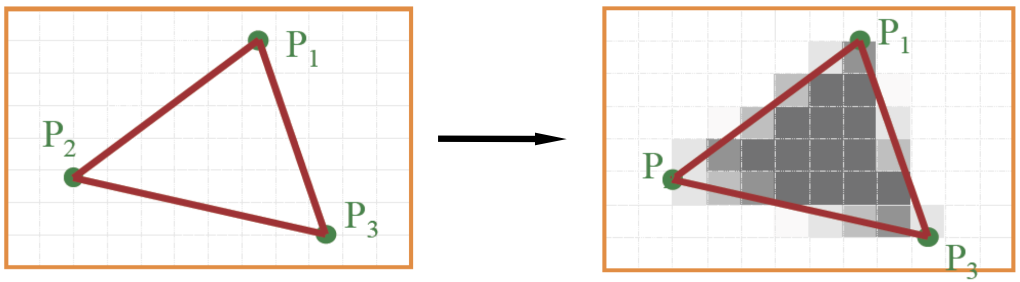
# Review

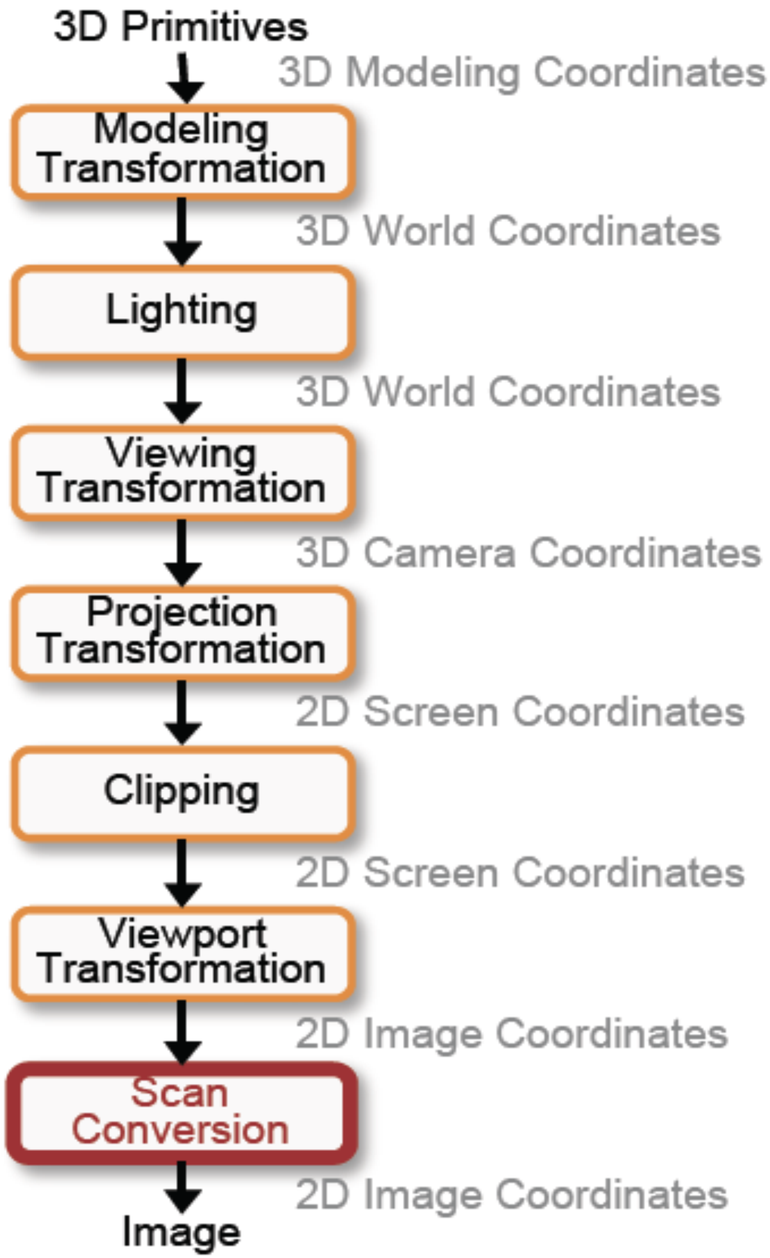
**Shading:**

- Determine a color for each filled pixel.

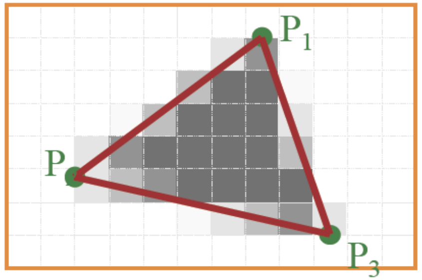
**Scan conversion:**

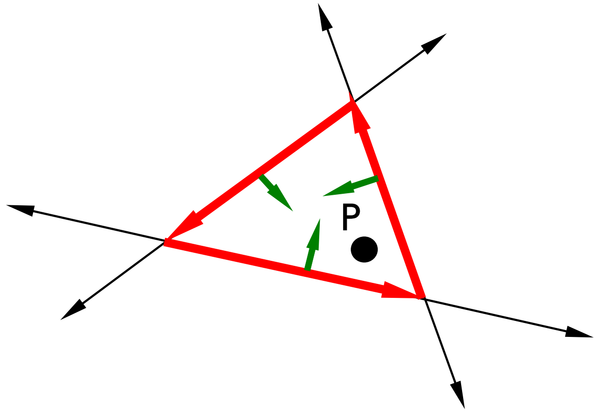
- Figuring out which pixels to turn on.
- Rendering an image of a geometric primitive by setting pixel colors.
- Example:
  - Filling the inside of a triangle.



# Triangle scan conversion




**Key idea:**

- Color all pixels ***inside*** triangle.

**Inside triangle test:**

- A point is inside a triangle if it is in the positive half-space of all three boundary lines.
  - Triangle vertices are ordered counter-clockwise.
  - Point must be on the left side of every boundary line.
- Recall that the implicit equation of a line:
  - On the line: $Ax+By+C=0$
  - On right: $Ax+By+C<0$
  - On left: $Ax+By+C>0$

```
void ScanCTriangle(Triangle T, Color rgba)
{
   for each pixel P(x,y)
      if inside(P,T)
            setPixel(x,y,rgba)
}
```

```
Boolean inside (Triangle T, Point P)
{
   for each boundary line L of  T {
      float dot = L.A*P.x+L.B*P.y+L.C*P.z;
            if dot<0.0 return FALSE;
   }
   return TRUE;
}
```

# Summary:

…:

- Raster display technology.
- Basic concepts: pixel, resolution, aspect ratio, dynamic range, image domain, object domain.
- Rasterization and direct illumination.
- Graphics primitives and OpenGL.
- Geometry representations: explicit, parametric and implicit forms.
- Rasterization algorithms for straight line segments, circles and ellipses.
- Rasterization algorithms for triangles and polygons.
- Rasterization versus shading.