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Energy-aware JPEG image compression: A multi-objective approach



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ABSTRACT

Customer satisfaction is crucially affected by energy consumption in mobile devices. One of the most energy-consuming parts of an application is images. This paper, first, investigates that there is a correlation between energy consumption and image quality as well as image file size. Therefore, these two can be considered as a proxy for energy consumption. In the next step, we focused on proposing a multi-objective strategy to enhance image quality and reduce image file size based on the quantisation table (OT) in IPEG image compression. To this end, we have used two general multi-objective approaches: scalarisation and Pareto-based. In this paper, we embed our strategy into five scalarisation algorithms, including energy-aware multi-objective genetic algorithm (EnMOGA), energy-aware multi-objective particle swarm optimisation (EnMOPSO), energy-aware multi-objective differential evolution (EnMODE), energy-aware multi-objective evolutionary strategy (EnMOES), and energy-aware multi-objective pattern search (EnMOPS). Also, two Pareto-based methods, including a non-dominated sorting genetic algorithm (NSGA-II) and a reference-point-based NSGA-II (NSGA-III) are used for the embedding scheme, and two Pareto-based algorithms, EnNSGAII and EnNSGAIII, are presented. With our proposed scalarisation method, user's preferences can be set before starting the optimisation process and the algorithm generates only one solution based on the preference, while our Pareto-based approaches generate a set of solutions so that a user can select one of the preferred solutions after the optimisation process.

Experimental studies show that the performance of the baseline algorithm is improved by embedding the proposed strategy into metaheuristic algorithms. In particular, EnMOGA, EnMOPS, and EnNSGA-II can perform competitively, among others. From the results, the baseline algorithm in all cases and in comparison to all algorithms yields the worst results. Among the scalarisation methods, EnMOGA and EnMOPS can achieve the first rank in 6 and 7 out of 13 cases and the second rank in 7 and 5 cases in terms of objective function. Also, EnMOES achieved the fifth or worst rank among the scalarisation algorithms. Regarding the Pareto-based algorithms, the table shows that EnNSGAII outperforms EnNSGAIII in 10 out of 13 cases in terms of hyper-volume measure, while it fails in 3 cases. Furthermore, we statistically verify the proposed algorithm's effectiveness based on the Wilcoxon-signed rank test. Finally, a sensitivity analysis of the parameters is provided. The source code for reproducing the results is available in: https://github.com/SeyedJalaleddinMousavirad/ MultiobjectiveIPEGImageCompression.

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1. Introduction

Mobile devices such as smartphones and tablets are ubiquitous and receiving much attention for their energy efficiency since customer satisfaction relies heavily on battery uptime. In addition, battery uptime plays a crucial role for developers since anomalous draining usually warrants negative app store ratings [1].

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https://doi.org/10.1016/j.asoc.2023.110278 1568-4946/© 2023 Elsevier B.V. All rights reserved. In recent years, several studies [2–7] have concentrated on documenting energy-aware programming trends in the context of Android, the leading mobile ecosystem, and finding better alternatives. But images have not been seriously discussed, while they are one of the most important components of mobile software, particularly in games.

JPEG (Joint Photographic Experts Group) format is the most commonly used method of compression for digital images, and is based on the Discrete Cosine Transform(DCT) [8]. The process of JPEG compression is started with representation of the original uncompressed image in YCbCr colour space, where Y, Cb and Cr indicate luminance, blue and red chrominance components, respectively; and each component is handled independently. We

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shall simply use the luminance component, Y, for the sake of simplicity, while for other components, the process is the same. The image's component Y is divided into 8×8 blocks, each of which is separately modified. Before using the DCT, the 8×8 blocks are zero-shifted by deducting 128 from the element values. Then, each modified block is quantised. The primary mechanism for compression, quantisation, also results in information loss due to the representation of the DCT coefficients. Each block may be effectively entropy encoded after quantisation [9], with no information being lost in the process.

The quantisation table (QT) plays a crucial role in the JPEG image compression. Annex K variant [10], the most important variant of JPEG implementation, employs two quantisation tables, called luminance quantisation table (LQT) and chrominance quantisation table (CQT). The main responsibility of these two is to quantise the DCT coefficient blocks of luminance and chrominance elements, respectively. The process of finding proper values for both quantisation tables is a challenging and difficult task since each image needs its own table, although most implementations use a conventional value for the tables.

To design the best quantisation table, meta-heuristic algorithms (MA) such as genetic algorithm (GA) [11] and particle swarm optimisation(PSO) [12] can be used. MAs are iterative, stochastic, and problem-independent algorithms that solve an optimisation problem by using a number of operators to guide the search process. Also, they can provide a close to optimal solution, but they are not able to guarantee a global optimum solution. In recent years, different MAs are applied for the construction of the JPEG QTs [13–16].

While images are one of the primary sources of energy consumption in smartphones, it is challenging to measure the amount of energy consumed for a specific operation in a typical image. Most of the current methods in the literature can measure the power of a battery or, at best, for a particular application [17]. To tackle this, we used the energy profiler of Android Studio and Plot Digitiser software, manually and not in an automatic way, to verify that image quality and file size play a crucial role in the energy consumption of an application. In other words, smaller file sizes and lower image quality consume less energy. A developer has two main goals in selecting an image: 1) they tend to select an image with high quality, and 2) they prefer to choose an image with smaller file size. As a result, there are two conflicting criteria for a mobile developer. Since an operation's energy for an image cannot be calculated as a straightforward process, file size and quality can act as a proxy for energy consumption.

These two criteria, file size and image quality, are two conflicting objectives. Therefore, multi-objective metaheuristic optimisation (MOMO) algorithms can be used to tackle this issue. MOMO addresses optimising a problem based on two or more conflicting objective. There are two general approaches for solving a multi-objective problem, namely, scalarisation and Pareto-based approaches [18]. Scalarisation approaches solve a multi-objective problem by converting it into a single-objective problem, while Pareto-based approaches generate a set of optimal solutions.

From another perspective, MOMO algorithms can be performed by a priori, interactive, or a posteriori methods. In the a priori methods, the users set their preferences prior to conducting the optimisation algorithm. As a result, a priori multi-objective optimisation does not yield several solutions, but rather only one compromise option. The scalarisation methods incorporate multi-objective functions into one single scalar objective function and can be solved by using a single objective metaheuristic algorithm. To the best of our knowledge, there is no scalarisation method, in the literature, for multi-objective JPEG image compression. A posteriori approach for multi-objective optimisation generates a set of solutions (instead of only one solution), from which the user can select the preferred solution. One of the most important a posteriori approaches is the non-dominated sorting genetic algorithm (NSGA-II) [19]. NSGA-II tries to find the optimal set of solutions by using several concepts such as non-dominated sorting, elite preserve operator, and crowding distance. This algorithm has shown competitive performance in solving complex multi-objective optimisation problems [20-22]. Later, a reference-point based non-dominated sorting genetic algorithm (NSGA-III) [23] is introduced by introducing a pre-defined set of reference points to guarantee diversity in the solutions, which has shown an acceptable performance in different types of optimisation problems [24,25]. From the literature, we can see that [26] proposed a set of solutions instead on only one single solution based on the NSGA-II approach. They compared NSGA-II with some standard algorithms such as standard JPEG and indicated that this approach can provide satisfactory QTs. In interactive methods, the user has a chance to indicate preferences, while multiple solutions are created during the optimisation process. There are several general approaches to interactive methods. For instance, calculating ideal and approximated nadir objective vectors and indicating them to a user is one of the well-known methods. Another method is to generate a new Pareto optimal solution(s) in terms of the new preferences.

This paper proposes an energy-aware JPEG Image compression strategy. The main contributions of this paper are as follows:

- We investigate, based on an energy profiler, that there is a high correlation between energy consumption and image quality. Such a condition is also valid for image file size.
- We propose a multi-objective strategy for handling both image file size and image quality.
- To the best of our knowledge, there is no a priory scalarisation method in the multi-objective JPEG image compression. Therefore, first, the proposed multi-objective strategy is embedded into five a priory scalarisation methods including a genetic algorithm (GA), differential evolution (DE), particle swarm optimisation (PSO), evolutionary strategy (ES), and pattern search (PS). Therefore, five scalarisationbased multi-objective techniques for JPEG image compression are introduced, namely, EnMOGA, EnMODE, EnMOPSO, EnMOES, and EnMOPS. As a result, a user, such as a developer, can set a preference before starting the optimisation process and the algorithm produces only one solution based on the preference.
- We also embed the proposed strategy into two well-known a posteriori Pareto-based approaches, the non-dominated sorting genetic algorithm (NSGA-II) [19] and referencepoint-based non-dominated sorting genetic algorithm (NSGA-III) [23]. As a result, two Pareto-based techniques are introduced here, namely, EnNSGAII and EnNSGAIII. As a result, a user, such as a developer, can select one of the generated images based on the priorities after the optimisation process.
- Generally speaking, not only the proposed strategy for JPEG image compression is novel, but also most algorithms selected are not used and assessed in JPEG image compression. For instance, adaptive PSO, pattern search and NSGA-III have not been used so far for JPEG image compression.
- To the best of our knowledge, there is no research on the computational complexity of population-based JPEG image compression. This paper presents the computational complexity of all algorithms.
- We provided an extensive set of experiments for validating the algorithms.

This paper is organised as follows: Section 2 provides a comprehensive review on the literature review. Section 3 explains briefly some challenges in the paper, while Section 4 introduces background knowledge. Section 5 explains the metaheuristic algorithms used in the paper. The proposed algorithms are introduced in Section 6, whereas we provide a set of extensive experiments in Section 7. Finally, the paper is concluded in Section 8.

2. Literature review

To the best of our knowledge, no research investigates the relationship between energy consumption and image characteristics, but little research has been done on finding the proper values for QTs by using meta-heuristic algorithms. Therefore, this section aims to provide a holistic review of the recent works on the JPEG QTs created by meta-heuristic algorithms.

In one of the first efforts to use MA for the construction of the JPEG OTs, [15] proposed a GA algorithm to find the quantisation table so that the chromosome is an array of size 64, while the objective function is the mean square error between the original image and the compressed image. In another work, [27] incorporated GA to design a JPEG image quantisation table to compress iris images in iris recognition systems. [28] represents each OT as a candidate solution and found the optimal values for QT based on the GA algorithm and for medical images. To enhance the performance of GA, [29] employed a real-coded quantum GA (RCQGA) for generating the optimal QT. RCQGA is more beneficial in the optimisation process compared to the standard GA since good candidate solutions are less likely to disappear after subsequent iterations. In another study, [26] proposed a set of solutions instead of only one single solution based on a multi-objective approach. They compared their algorithm with some standard algorithms such as standard JPEG and showed that this approach can provide satisfactory QTs. [30] proposed a knowledge-based GA to find the quantisation table. To this end, image characteristics and knowledge about image compression are integrated into the GA algorithm. For instance, the candidate solutions are randomly generated, but values in the top left subtable in QT are less than in the bottom right sub-table since, due to the knowledge domain, the value of the QT at the top left should be less than that at the bottom right of the QT.

Differential evolution (DE) [31] is another evolutionary-based MA used for JPEG image compression. [13] compared DE with GA algorithms in optimising the QT for the JPEG baseline algorithm. They showed that DE could outperform canonical GA. Another study [32] proposes a knowledge-based DE to improve the performance of DE. The knowledge embedded in this research was similar to the earlier research [30]. To lessen the computation time of the DE algorithm,[33] employed a problem approximation surrogate model (PASM) to aid the DE algorithm in finding the optimal values of QT. PASM reduced the computation time of the DE algorithm. Also, they embedded the domain knowledge to improve performance.

From the literature, we can also observe some other MA algorithms for designing QTs in JPEG. For instance, the Firefly algorithm (FA) [34] is employed to optimise the QT in JPEG image compression based on a similarity-based objective function. They compared the FA algorithm with standard JPEG image compression and indicated that FA can provide satisfactory results. A similar work has been done using firework algorithm [35]. PSO and dual SA are employed to find the optimal QTs in [36], while they optimised a Lagrangian Rate-Distortion objective function and a fixed quality expected rate gain. In one of the recent works, simulated annealing (SA) [37] as a single-solution-based MA is employed to optimise the QT values. [38] employs a SA algorithm to strike a balance between the computational time and image-specific optimality by offering a novel idea of texturing mosaic images. The SA algorithm is utilised on texture mosaic images to find the best QT for each texture category instead of optimising a single image or a set of sample images. To learn the texture features and predict the texture distribution of a new image, they employed the pre-trained VGG-16 CNN model. They then combine the optimal texture tables to produce an ideal QT tailored to the particular image.

We can also see that few researchers boost the performance of JPEG image compression by fusing several algorithms. For instance, [39] proposed a combination of FA and teaching–learningbased optimisation (TLBO) to select the QT. To this end, TLBO is added to the FA algorithm to explore the search space more effectively.

3. Key challenge

One of the main challenges in calculating the energy consumption of an application is figuring out how to do it. Some research uses hardware devices for this purpose [40], which is hard to set up. They calculate the energy consumption of the battery and not an android application. Some others try to estimate the energy profile of an android application, which is not straightforward to do as well [17].

This section investigates the effect of image file size and image quality on energy consumption. To this end, we used an energy profiler in the Android Studio software [41] and a plot digitiser [42] to estimate energy consumption. Plot digitisers are tools to convert a specific curve to digitised numbers. To this end, first, the digitiser should be calibrated for the curve (here between 0 and 2000). In other words, we should specify the minimum and maximum values on the *y*-axis. In this case, the minimum and maximum values are not critical since we only need a comparison between the results (and not an exact number for energy consumption).

To design the experiment, we compress an image at different levels (90%, 70%, 50%, 30% and 10%), and the energy consumption (EC) for each image is calculated. To this end, an image loading program is written, located in a loop with 1000000 iterations and then, the energy profile is achieved for this during five independent runs (Fig. 1). Then, the energy profiler yielded is fed to the plot digitiser to convert it to digit numbers, and for each run, the total energy consumption is estimated. Finally, the average over five runs is obtained as the EC measure. The size and quality (based on PSNR) are also reported for each image. The results can be seen in Table 1. From the table, we can conclude that:

- 1. By decreasing the image compression level, the EC is decreased as well.
- 2. By decreasing the image compression level, as expected, the file size is decreased as well.
- 3. By decreasing the image compression level, as expected, the image quality deteriorated.
- 4. By decreasing the image size, the image quality is also reduced.

All in all, reducing the compression level reduces the file size, image quality, and energy consumption, while the developer tends to use higher image quality and smaller file size. Therefore, these two objectives, image quality and image size, are in conflict.

The correlation between image size and image quality (based on the information available in Table 1) is demonstrated in Table 2 to show this contradiction. This table clearly verifies a conflict between higher image quality and smaller file size since the correlation is a high positive number, close to 1.

In conclusion, file size and image quality can be considered proxies of energy consumption. While developers tend towards



Fig. 1. Energy profiler for an image loading program in 5 independent runs.

Table 1

Effect of image size and image quality on the energy consumption.

Level	EC	File size (Mb)	PSNR
Original	2769.52	3.07	∞
90	2734.73	1.90	38.6913
70	2682.73	1.50	37.5774
50	2638.61	0.98	35.1707
30	2566.47	0.66	33.0362
10	2511.69	0.27	28.3434
10	2511.69	0.27	28.3434

Table 2

Correlation between EC criterion and file size with other.

Correlation	File size	PSNR
EC	0.9433	0.9754
File size	1	0.9615

smaller file sizes and higher image quality, these two objectives conflict with each other since higher image quality will increase the file size and energy consumption. As a result, it is necessary to strike a balance between image quality and file size.

4. Preliminaries

This section provides background knowledge of our research including multi-objective optimisation and JPEG image compression.

4.1. Multi-objective optimisation

Multi-objective optimisation (MO) is the process of finding the minimum or maximum of two conflicting objective functions. Without loss of generality, a multi-objective optimisation problem (MOP), formally, can be stated as a minimisation problem as

$$\begin{array}{ll} \text{Minimise} \quad F(x) = (f_1, f_2, \dots, f_M) \\ & \text{subject} \quad \text{to} \quad x \in \Omega \end{array}$$
(1)

where Ω is the decision space, and $F : \Omega \to \mathbb{R}^M$ is the objective function in which *M* is the number of different real-valued objective functions, and \mathbb{R}^M shows the objective space.

There are two general metaheuristic approaches for tackling multi-objective optimisation, scalarisation and Pareto-based approaches [18]. Scalarisation approaches solve a multi-objective problem by converting it into a single-objective problem, while Pareto-based techniques find a set of optimal solutions.

Scalarisation approaches incorporate multi-objective functions into one single scalar objective function as

$$G(x) = w_1 f_1(x) + w_2 f_2(x) + \dots + w_M f_M(x)$$
(2)

The real-valued positive weights, w_i , i = 1, 2, ..., M, indicate the performance priority. A larger weight shows that the corresponding objective function has a higher priority than the objective function with a smaller weight. When the priority of objective functions is not clear in advance, one of the most common methods is to use Equal Weights [18], in which the weights are given by

$$w_i = \frac{1}{M} \tag{3}$$

where i = 1, 2, ..., M.

After scalarisation, all single-solution-based metaheuristic algorithms such as GA [11], PSO [43], and DE [31] can be used to find the optimal solution.

There is usually no single solution that can simultaneously minimise all the objective functions since the objectives are inherently competing. To tackle this, a set of optimal solutions, called *Pareto optimal solutions*, can be defined, with corresponding localisation in the objective space called the *Pareto front*.

In single-objective optimisation, the superiority of one solution over another can be easily obtained by comparing the objective functions, while the quality of a solution can be achieved by the concept of dominance in multi-objective optimisation.

Definition 1 (*Pareto Dominance*). A solution x_1 dominates another solution x_2 (denoted by $x_1 \prec x_2$) if and only if:

- 1. $\forall i \in \{1, 2, ..., M\}$: $f_i(x_1) \leq f_i(x_2)$, where *M* is the number of objective functions. In other words, in all objective functions, solution x_1 should not be worse than x_2 .
- 2. $\exists j \in \{1, 2, ..., M\}$: $f_i(x_2) < f_i(x_1)$; meaning that solution x_1 is strictly superior to solution x_2 in at least one objective function.

Definition 2 (*Non-dominated Solution*). A solution x_1 is called Pareto optimal solution or non-dominated solution if it is not dominated by other solutions in the whole search space. It can be mathematically defined as

$$\nexists x_2 \in X : x_2 \prec x_1 \tag{4}$$

Fig. 2 indicates non-dominated solutions among other solutions in a bi-objective minimisation problem. f_1 and f_2 are two



Fig. 2. Non-dominated solutions in a bi-objective optimisation problem.



Fig. 3. The main components of the JPEG image compression.

conflicting objectives which should be minimised simultaneously. From the figure, x_3 has a lower value than x_1 in both objective functions. Therefore, we can say that x_1 is dominated by x_3 . In other words, x_3 is a non-dominated solution. In addition, x_2 , x_4 , x_5 and x_6 are also non-dominated solutions since there is no other solution that dominates them in both objective functions.

Definition 3 (*Pareto Front*). The set of all non-dominated solutions is called Pareto optimal set (PS), which is stated as

$$PS = \{ u \in X | \exists v \in X, u \prec v \}$$
(5)

The Pareto front (PF) corresponds to the Pareto optimal set in the objective space, and is denoted as

$$PF = \{F(x)|x \in PS\}$$
(6)

4.2. The JPEG image compression

Fig. 3 shows the main components of JPEG image compression. The encoder is responsible for converting the original image into the JPEG compression variant of the original image, while the reverse task is carried out by the decoder. In the following, we explain the main components in more detail.

4.2.1. DCT and IDCT components

The source image is first divided into 8×8 blocks. Then, the values of the blocks are shifted from $[0, 2^p - 1]$ to $[-2^{p-1}, 2^{p-1} - 1]$, in which *p* is the number of bits per pixel (in the baseline JPEG compression, p = 8). Each block of 8×8 pixels can be seen as a vector with a size of 64×1 , which should be fed into the Discrete Cosine Transform (DCT) [44] component. The DCT block decomposes the input signal into 64 basis-signal amplitudes, called DCT coefficients. Mathematically, the DCT can be expressed as

$$F(u, v) = \frac{1}{4}c_u c_v \left[\sum_{x=0}^{7} \sum_{y=0}^{7} f(x, y) \cos\left(\frac{(2x+1)u\pi}{16}\right) \cos\left(\frac{(2y+1)v\pi}{16}\right)\right]$$
(7)

where

$$c_r = \begin{cases} \frac{1}{\sqrt{2}} & r = 0\\ 1 & r > 0, \end{cases}$$
(8)

The DC coefficient is the one corresponding to u, v = 0, while the other 63 coefficients are known as the AC coefficients.

Inverse DCT (IDCT) is the reverse of DCT component to reconstruct the original image, which is stated as

$$F(x, y) = \frac{1}{4} c_u c_v \left[\sum_{u=0}^{7} \sum_{v=0}^{7} f(u, v) \cos\left(\frac{(2x+1)u\pi}{16}\right) \cos\left(\frac{(2y+1)v\pi}{16}\right) \right]$$
(9)

In the absence of the quantisation step, the original 64-point signal is precisely restored.

4.2.2. The quantisation and dequantisation components

The quantisation step works based on a 64-element quantisation table, which should be known in advance. Each table entry defines the step size of the quantiser for its related DCT coefficient and belongs to [1,255]. Quantisation aims to achieve compression while maintaining image quality by removing information that is not visually important.

The quantisation component is defined as

$$L(u, v) = round\left(\frac{F(u, v)}{Q(u, v)}\right),$$
(10)

where L(u, v) are the quantised DCT coefficients, F(u, v) are the DCT coefficients, Q(u, v) indicates the corresponding element of the quantisation table, and round(x) is the closets integer number to x. It is worth mentioning that the larger the value of Q(u, v), the larger the information loss.

The de-quantisation component of the decoder reverses the quantisation process to recreate a rough estimate of F(u, v) from L(u, v) as

$$F(u, v) = L(u, v) \times Q(u, v), \tag{11}$$

This step plays a crucial role in the process of JPEG compression since the quantisation table generates a loss of information. Thus, it is necessary to establish the quantisation table to strike a compromise between compression effectiveness and reconstructed image quality.

4.2.3. Symbol coding

The 63 AC coefficients of the 8×8 block are handled independently from the DC coefficient after quantisation. The Differential Pulse Code Modulation (DPCM) is used to encode the DC coefficient as

$$DIFF_i = DC_i - DC_{i-1} \tag{12}$$

where DC_i and DC_{i-1} are the DC coefficients for the current 8×8 block and the prior 8×8 block, respectively.

In order to format the quantised 63 AC coefficients for entropy coding, a zigzag scan [45] can be used. After the zigzag scan, the AC coefficients show diminishing variances and rising spatial frequencies.

4.2.4. Entropy coding

After the quantisation process, there are often a few nonzero and several zero-valued DCT coefficients. Entropy coding's goal is to compress the quantised DCT coefficients by making use of their statistical properties. The baseline technique used by JPEG is the Huffman coding, which employs two DC and two AC Huffman tables for the luminance and chrominance DCT coefficients, respectively [45].

5. Algorithms

Due to the introduction of a vast and varied range of metaheuristic techniques in the literature, it is evident that we cannot analyse all of them. Also, the main focus of this paper is not benchmarking all algorithms but introducing a general strategy for multi-objective JPEG image compression. Therefore, for our study, we have chosen a variety of state-of-the-art algorithms. In the following, we briefly outline the selected algorithms, while the cited publications are referred to for further details.

5.1. Scalarisation methods

- Genetic algorithm (GA) [11] is the oldest metaheuristic algorithm, and includes three significant operators: selection, crossover, and mutation. The selection operator is responsible for selecting candidate solutions who contribute to the next generation's population. The information from the parents is integrated into the crossover operator, while random modifications are made to one or more components of a potential solution in the mutation operator. Based on the "survival of the fittest" premise, solutions are transferred from one iteration to the next.
- Differential evolution (DE) [31] is another metaheuristic algorithm including three main operators, mutation, crossover, and selection. Mutation creates candidate solutions based on the differences among candidate solutions as

$$v_i = x_{r1} + SF(x_{r2} - x_{r3}), \tag{13}$$

where SF signifies a scaling factor, and x_{r1} , x_{r2} , and x_{r3} are three distinct randomly selected candidate solutions from the current population, and v_i is called a mutant vector. Crossover is responsible for integrating the mutant vector with a target vector selected from the current population. Eventually, a candidate solution is selected by a selection operator depending on its quality.

• Particle swarm optimisation [12] is a swarm-based optimisation approach, and its updating scheme is based on the best position found for each candidate solution and a global best position. The velocity vector of a particle is updated as

$$v_{t+1} = \omega v_t + c_1 r_1 (p_t - x_t) + c_2 r_2 (g_t - x_t), \tag{14}$$

where *t* shows the current iteration, x_t is the current position, r_1 and r_2 are random numbers generated from a uniform distribution in the range of [0, 1], p_t is the personal best position, and g_t indicates the global best position. Then, a candidate solution is updated as

$$x_{t+1} = x_t + v_{t+1},\tag{15}$$

• Evolutionary strategy [46] is a metaheuristic algorithm where each offspring is generated based on a Gaussian random number as

$$x_{new} = x_{old} + N(0, \sigma^2), \tag{16}$$

where $N(0, \sigma^2)$ is a Gaussian random number with mean 0 and variance σ^2 . Then, competition should be done for each individual and finally, the best individuals transfer to the next generation.

• Pattern search [47] is a simple yet effective optimisation algorithm that, in an iterative manner, combines exploratory and pattern moves to find the best solution to a problem. The exploratory move tries one direction, and if that does not work, it tries the other. In particular, it generates a new solution as

$$x^+ = x + \rho, \tag{17}$$

where x^+ is the new solution based on the current solution x, and ρ is called the step size or exploratory radius. If



Fig. 4. Non-dominated sorting procedure.

this move cannot improve the current solution, it attempts another direction

$$x^+ = x - \rho, \tag{18}$$

If the moves in all directions fail, then the radius is halved.

5.2. Pareto-based techniques

5.2.1. Non-dominated sorting genetic algorithm II

The Non-Dominated Sorting Genetic Algorithm (NSGA-II) [19] is one of the state-of-the-art approaches for Pareto-based multiobjective optimisation. NSGA-II is based on four basic operators, including, Non-Dominated Sorting, Elite Preserving Operator, Crowding Distance, and Selection Operator, which are described below in more detail.

Non-Dominated Sorting: The notion of Pareto dominance is used in this process to sort the population members. In the first step, the non-dominated members of the initial population are assigned to the first rank. These top-ranked individuals are subsequently put in the first front and eliminated from the current population. The remaining population members are then subjected to the non-dominated sorting technique. The remaining population's non-dominated individuals are given the second rank and positioned in the second front. This procedure continues until all population members are distributed across various fronts in accordance with their rankings, as seen in Fig. 4.

Elite Preserving Operator: Elite solutions are maintained by being immediately passed on to the next generation as part of an elite preservation strategy. In other words, the non-dominated solutions identified in each generation transfer to the next generations until some solutions dominate them.

Crowding Distance: The crowding distance determines the density of solutions around a specific solution. It is the average distance between two solutions along each of the objectives on each side of the solution. When two solutions with varying crowding distances are compared, the solution with the greater crowding distance is assumed to be present in a less crowded area. The crowded distance of the *i*th solution is computed based on the average side-length of the cuboid (Fig. 5). Mathematically, the crowding distance is defined as

$$CD_i = \sum_{j=1}^k \frac{f_j^{i+1} - f_j^{i-1}}{f_j^{\max} - f_j^{\min}},$$
(19)

where f_j^i shows the *j*th value of an objective function for the *i*th solution, f_j^{max} and f_j^{min} signify the maximum and minimum values of *j*th objective function among the current population, and *k* is the number of objective functions.

Selection Operator: A crowded tournament selection operator is used to choose the population for the next generation. This operator selects the population based on the rank of the population members and the crowding distances between them. The following rules apply when choosing one of two population members to represent the next generation: 1) If the two population members are of different ranks, the higher rank one is chosen; and 2) If the two population members are of the same rank, the member with the greater crowding distance is chosen.

Procedure: The NSGA-II algorithm starts by creating an initial population P_t of size N. Following crossover and mutation operations on the population, P_t , a new population, Q_t , is produced. Then, the non-dominated sorting operation is carried out on the new population, R_t , created by combining the populations P_t and Q_t . The R_t population members are then divided into several fronts based on their non-dominance levels.

The next step is to choose *N* candidate solutions from R_t in order to produce P_{t+1} . If the size of the first front is greater than or equal to *N*, only *N* members are chosen from the least crowded area of the first front to create P_{t+1} . The members of the first front are directly moved to the next generation if the size of the first front is more than *N*, and the remaining members are taken from the second front's least crowded area and added to P_{t+1} . The process is repeated for the subsequent fronts until the size of P_{t+1} equals *N*, if the size of P_{t+1} is still less than *N*. Following the same process, the populations P_{t+2} , P_{t+3} , P_{t+4} ...,for subsequent forms are created until the stopping criterion is not met. Fig. 6 shows the NSGAII procedure visually.

5.3. Reference-point based non-dominated sorting genetic algorithm

The basic framework of Reference-point Based Non-dominated Sorting Genetic Algorithm (NSGA-III) [23] is similar to NSGA-II, but with significant modifications to its selection process. Unlike NSGA-II, NSGA-III adaptively updates several widely used reference points, which aids in maintaining diversity among population members.

As previously mentioned, the NSGA-III employs a pre-defined set of reference points to guarantee diversity in the solutions produced. The standard NSGA-III algorithm benefits from Das



Fig. 6. The NSGAII procedure.

Rejected

and Dennis's [48] approach which assigns points to a normalised hyper-plane. The total number of reference points (H) in an M objective problem by P division can be calculated as

R

$$H = \begin{pmatrix} M+P-1\\ P \end{pmatrix}$$
(20)

For instance, in a problem with three objectives (M = 3), the reference points are made on a triangle whose apex is at (1, 0, 0), (0, 1, 0), and (0,0,1). For each objective axis, four divisions (P = 3) will result, and therefore a total of 10 reference points (Fig. 7).

NSGA-II utilises the crowding distance to pick the remaining members after non-dominated sorting, while with NSGA-III, the reference points are used to select the remaining members. To do this, the range of objective values and reference points are first normalised to be the same. Afterwards, the orthogonal distance between each reference line and a member of P_t is calculated. The reference point with the shortest orthogonal distance is then used to associate the member with.

The number of individuals who are connected to each reference point, known as the niche count for each reference point, is then computed for further analysis. Then, the reference point with the lowest niche point is found and the member from the last front that is associated with it should be included in the final population. It is important to note that a reference point need not have any population members linked with it and may have one or more related population members. For the *j*th reference point, the number of population members that are associated with each reference point is counted and denoted as niche count (ρ_j). NSGA-III employs a niche-preserving operator as follows. First, a reference point set, $J_{min} = j : argmin_i\rho_i$ is defined, including a



Fig. 7. 3D plot of the 10 reference points with p = 3.

minimum ρ_j . When there are several of these reference points, one $(j^* \in j_{\min})$ is selected at random. If $\rho_{j^*} = 0$ (meaning that there is no associated member to the reference point j), two scenarios can happen. First, the reference point j is already connected to one or more of the members in front of F_1 . In this case, the one closest to the reference line perpendicularly is added to P_{t+1} . Second, the front F_1 does not have any members linked to

the reference point. In this case, the reference point is not taken into account anymore for the current generation.

In the case of $\rho_j \ge 1$ (indicating that one member associated with the reference point exists in P_t/F_L), a randomly selected number from front F_L that is associated with the reference point ρ_j is added to P_{t+1} . After updating the niche counts, the process is repeated *K* times in total to bring P_{t+1} 's population size to *N*.

6. Proposed methods

This paper proposes a general strategy for multi-objective optimisation of JPEG implementation. As a result, it can be used with any optimisation algorithm. To this end, we embed our proposed scheme into five scalarisation algorithms, GA, DE, PSO, ES, and PS, and two Pareto-front-based algorithms, NSGA-II and NSGA-III. First, we define the solution representation and objective function in the following. Then, by embedding the proposed strategy within seven backbone optimisation algorithms, we obtain seven new algorithms, namely EnMOGA, EnMODE, EnMOPSO, EnMOES, EnMOPS, EnNSGAII, and EnNSGAIII, respectively.

6.1. Solution representation

Our proposed algorithm aims to find multi-objective optimal quantisation tables, including luminance quantisation table (LQT) and chrominance quantisation table (CQT). To this end, each 8by-8 quantisation table is reshaped to a 1-by-64 vector, and then both are concatenated. Therefore, the representation proposed in this paper is a vector of dimension 128 as

$$x = [LQT_{1,1}, \dots, LQT_{8,8}, \dots, CQT_{1,1}, \dots, CQT_{8,8}]$$
(21)

where $LQT_{i,j}$ and $CQT_{i,j}$ show the corresponding element in the location (i, j) in the LQT matrix and the CQT matrices. In other words, the first 64 entries are positive integer numbers in $[0, 2^p - 1]$ (where *p* is the number of bits indicating a pixel, in our case p = 8) for the LQT table, while the remaining elements are reserved for the CQT table.

6.2. Objective functions

This paper introduces two main objective functions, file size and image quality. One is to be minimised (file size) and the other to be maximised (image quality). To this end, first, the JPEG image should be achieved using the corresponding candidate solution for a typical image. The first objective function is file size defined as

$$FS_{obj} = \frac{FS_{JPEG}}{FS_{org}}$$
(22)

where FS_{JPEG} is the file size for image after JPEG compression process, and FS_{org} is the image size for the original image. Lower FS_{obj} shows a higher capability in the compression process.

The second objective function is Peak Signal to Noise (PSNR), as one of the most common measures for assessing image quality, which is computed as

$$PSNR = 20 \log_{10}(255/RMSE),$$
 (23)

where *RMSE* is the root mean squared error which is calculated as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (I(i,j) - \hat{I}(i,j))^2}{MN}},$$
(24)

where *M* and *N* are the image dimensions, and *I* and \hat{I} are the original and the compressed images. A higher PNSR value indicates better performance.

Algorithm 1 EnGAMO algorithm in the form of pseudo-code.

- 1: **Inputs:** L/U: lower/upper bound; N_{pop} : population size; N_{var} : number of variables; NFE_{max} : maximum number of function evaluations; *prob*: the probability of the crossover, η : the distribution index.
- 2: Initialise population of N_{pop} candidate solutions using the representation introduced in Section 6.1.
- 3: Calculate objective function values (OFV) of all candidate solutions (Section 6.2).
- 4: x^* = best candidate solution in the initial population
- 5: $NFE = N_{pop}$
- 6: *iter* = 0
- 7: while $NFE <= NFE_{max}$ do
- 8: iter = iter + 1
- 9: Perform Tournament selection (Section 6.3.1).
- 10: Perform SBX crossover (Section 6.3.1).
- 11: Perform Polynomial mutation (Section 6.3.1).
- 12: Calculate objective function values of all new candidate solutions (Section 6.2).
- 13: Replace the old population by the new one.
- 14: x^+ = best candidate solution in the current population.
- 15: **if** OFV of $x^+ <$ OFV of x^* **then**
- 16: $x^* = x^+$
- 17: end if
- 18: $NFE = NFE + N_{pop}$
- 19: end while

Scalarisation approaches integrate the multi-objective functions into one objective function. Therefore, the objective function for the scalarisation methods is expressed as

$$F(x) = w_1 \cdot FS_{obj} + \frac{w_2}{PSNR}$$
(25)

where w_1 and w_2 are two used-defined parameters, indicating the importance of each objective function.

Pareto-based approaches can work on our two objective functions independently, so there is no need to combine two objective functions. As a result, the two objective functions for Pareto-based approaches are FS_{obj} and $\frac{1}{PSNR}$.

6.3. Embedding within scalarisation approaches

6.3.1. EnMOGA Algorithm

The EnMOGA begins with forming a random initial population from a uniform distribution. Over various generations, new populations are created by applying crossover, mutation, and selection operators. The pseudo-code of the EnMOGA algorithm is given in Algorithm 1, while the components are briefly explained below.

Selection: We use tournament selection, which promotes quicker convergence. In tournament selection, the top candidate solutions are chosen from a random subset of the population for each tournament. The size of the tournament is determined by the number of participants in each tournament.

Crossover: We use Simulated Binary Crossover (SBX) [49] for the crossover operator. A binary notation can express real values, and then a point crossover can be performed. By using a probability distribution model of the binary crossover, SBX replicated this process. SBX benefits from two leading parameters, including the probability of a crossover and the distribution index (η).

Mutation: Polynomial Mutation [49] is used in this paper, which follows the same probability distribution as the SBX operator in the parent's vicinity. It also has the same parameters as the SBX operator.

Algorithm 2 EnMODE algorithm in the form of pseudo-code.

- 1: **Inputs:** L/U: lower/upper bound; N_{pop} : population size; N_{var} : number of variables; NFE_{max} : maximum number of function evaluations; CR: crossover rate.
- 2: Initialise population of N_{pop} candidate solutions using the representation introduced in Section 6.1.
- 3: Calculate objective function values of all candidate solutions (Section 6.2).
- 4: x^* = best candidate solution in the initial population
- 5: $NFE = N_{pop}$
- 6: *iter* = 0^{-1}
- 7: while $NFE <= NFE_{max}$ do
- 8: iter = iter + 1
- 9: Perform Dither operation (Section 6.3.2).
- 10: Perform Mutation operator (Section 5.1).
- 11: Perform Crossover operator (Section 5.1).
- 12: Calculate objective function values of all new candidate solutions (Section 6.2).
- 13: Perform Selection operator (Section 5.1).
- 14: x^* = best candidate solution in the current population.
- 15: end while

6.3.2. EnMODE algorithm

Since EnMODE is a population-based metaheuristic, it is started with a random initial population. It has three main operators, including, mutation, crossover, and selection. For them, we used the standard operators, described in Section 5.1. Also, EnMODE benefits from Dither [50], a deterministic scheme of randomisation of the scale factor *SF* (introduced in Section 5.1). Dither proposes selecting *SF* from the interval [0.5, 1.0] randomly for each generation. The pseudo-code of EnMODE is given in Algorithm 2.

6.3.3. EnMOPSO Algorithm

The EnMOPSO is based on the PSO algorithm. The updating strategy used here is similar to the standard PSO algorithm introduced in Section 5.1. Standard PSO uses two parameters, c_1 and c_2 . Here, both parameters are updated based on the way proposed in [51]. To this end, PSO has been placed in 4 states, including convergence, exploitation, exploration, and jumping out. In each state, one of the following operations should be performed.

- 1. Increasing c_1 and decreasing c_2 in an exploration state,
- 2. Increasing c_1 slightly and decreasing c_2 slightly in an exploitation State,
- 3. Increasing *c*₁ slightly and increasing *c*₂ slightly in a convergence state,
- 4. Decreasing c_1 and increasing c_2 in a jumping out state.

The evolutionary states estimation process is as follows.

1. Calculate the mean distance of each particle (d_i) with all other particles as

$$d_i = \frac{1}{N-1} \sum_{j=1, j \neq i}^{N} \sqrt{\sum_{k=1}^{D} (x_i^k - x_j^k)}$$
(26)

where *N* and D are population size and the number of dimensions, respectively.

2. Calculate the evolutionary factor as

$$ef = \frac{d_g - d_{\min}}{d_{\max} - d_{\min}}$$
(27)

Algorithm 3 EnMOPSO algorithm in the form of Pseudo-code.

- 1: **Inputs:** *L*/*U*: lower/upper bound; *N*_{pop}: population size; *N*_{var}: number of variables; *NFE*_{max}: maximum number of function evaluations; *G*: maximum number of iterations.
- 2: g = 1
- 3: $NFE = N_{pop}$
- 4: iter = 0
- 5: Initialise population of N_{pop} candidate solutions using the representation introduced in Section 6.1.
- 6: Calculate objective function values of all candidate solutions (Section 6.2).
- 7: Initialise *Gbest* as a candidate solution with the minimum value of the population.
- 8: Initialise *Pbest* to its initial position for each candidate solution.

9: while
$$NFE <= NFE_{max}$$
 do

- 10: iter = iter + 1
- 11: Estimate the evolutionary states of the algorithm and calculate evolutionary factor using Eq. (27).
- 12: Select one of 4 states, including convergence, exploitation, exploration, and jumping out to update the parameters.
- 13: Update ω using Eq. (28).
- 14: Calculate the particle's velocity according to Eq. (14).
- 15: Update particle's position according to Eq. (15).
- 16: Calculate objective function values of all new candidate solutions (Section 6.2).
- 17: Update Gbest and Pbest.
- 18: x^* = best candidate solution in the current population.
- 19: Update NFE
- 20: end while

where d_g means the distance for the global best position, and d_{max} and d_{min} are the maximum and minimum distances, respectively.

 Classify *ef* into one of four sets (based on the rules introduced in [51]), which represents the states of exploration, exploitation, convergence, and jumping out.

In addition to updating c1 and c2, ω also updates based on a Sigmoid function as

$$\omega(ef) = \frac{1}{1 + 1.5e^{-2.6ef}} \tag{28}$$

The EnMOPSO algorithm in the form of Pseudo-code is given in Algorithm 3.

6.3.4. EnMOES algorithm

EnMOES algorithm is inspired by evolutionary strategy [46], and includes two leading operators, namely mutation and selection. Mutation operator is performed using Eq. (16), while selection is based on objective function ranking. The EnMOES algorithm in the form of pseudo-code is given in Algorithm 4.

6.3.5. EnMOPS algorithm

EnMOPS works based on pattern search; therefore, it tries to find the optimal point by comparing, at each iteration, its value with a finite set of trial points. The Pseudo-code of EnPS-MO algorithm is given in Algorithm 5.

6.4. Embedding within Pareto-based techniques

This subsection presents how to embed the proposed multiobjective JPEG image compression strategy into two well-known Pareto-based techniques, NSGA-II and NSGAIII. Algorithm 4 EnMOES algorithm in form of Pseudo-code.

- 1: **Inputs:** L/U: lower/upper bound; N_{pop} : number of bids; N_{var} : number of variables; NFE_{max} : maximum number of function evaluations; σ : variance value for Gaussian distribution.
- 2: Initialise population of N_{pop} candidate solutions using the representation introduced in Section 6.1.
- 3: Calculate objective function values of all candidate solutions (Section 6.2).
- 4: x^* = best candidate solution in the initial population.
- 5: $NFE = N_{pop}$
- 6: *iter* = 0

7: while $NFE <= NFE_{max}$ do

- 8: iter = iter + 1
- 9: Select parents in a random manner.
- 10: Generate offspring using Eq. (16).
- 11: Calculate objective function values of all offspring (Section 6.2).
- 12: Select the best candidate solutions among the combination of offspring and parents.
- 13: x^* = best candidate solution in the current population.
- 14: end while

6.4.1. EnNSGAII algorithm

EnNSGAII algorithm is a Pareto-based technique and generates a set of solutions instead of a single solution. EnNSGAII employs the NGSA-II algorithm for the optimisation process introduced in Section 5.2.1. We have used the same operators for EnNS-GAII algorithm including non-dominated sorting, elite preserving operator, crowding distance, and selection operator. For the evolutionary step, we have used similar operators to the ones used in EnMOGA. In other words, we have used tournament selection, SBX crossover, and Polynomial mutation. Algorithm 6 presents the pseudo-code for the EnNSGAII algorithm.

6.4.2. EnNSGAIII algorithm

EnNSGAIII is similar to EnNSGAII except that it employs reference directions rather than crowding distance. Therefore, all operators we used for EnNSGAIII are similar to EnNSGAII. In other words, EnNSGAIII employs non-dominated sorting, elite preserving operator, tournament selection, SBX operator, and Polynomial mutation for the optimisation process. The EnNSGAIII algorithm in the form of pseudo-code is given in Algorithm 7.

6.5. Computational complexity

This section provides a computational complexity analysis of the proposed algorithms. Generally speaking, the computational complexity of an approach based on a metaheuristic algorithm depends on different criteria such as population size (N_{pop}) , objective function, the number of iterations (*I*), problem dimensions(*d*), and operators. The computational complexity of the operators in our problem is usually lower than the complexity of the objective function. As a result, the complexity of the operators can be ignored in most cases. In the following, first, we discussed the computational complexity of JPEG image compression since it is the main part of the objective function. Then, we explain the computational complexity of our proposed algorithms.

JPEG Image Compression: the computational complexity of JPEG image compression is mainly dependent on the components of the algorithm and the size of the input image. The computational complexity of colour space conversion is O(N), where N is the number of pixels in the input image. The DCT is performed on 8 × 8 blocks of pixels, and its computational

Algorithm 5 EnMOPS algorithm in the form of Pseudo-code.

- Inputs: L/U: lower/upper bound; N_{var}: number of variables; NFE_{max}: maximum number of function evaluations; ρ: step size
- 2: Generate a randomly candidate solution (x) using the representation introduced in Section 6.1.
- 3: Calculate objective function values (OFV) of the candidate solution (Section 6.2).
- 4: NFE = 1
- 5: *iter* = 0
- 6: while $NFE <= NFE_{max}$ do
- 7: iter = iter + 1
- 8: Generate one trial solution (x^+) using Eq. (17).
- 9: Calculate objective function values of the new trial solution (Section 6.2).
- 10: **if** x^+ is better than the current solution **then**
- 11: replace the current solution by x^+ .
- 12: else
- 13: Generate one trial solution (x^+) using Eq. (18).
- 14: Calculate objective function values of the new trial solution (Section 6.2).
- 15: **if** x^+ is better than the current solution **then**
- 16: replace the current solution by x^+ .
- 17: end if
- 18: end if
- 19: **if** x^+ is worse than the current solution **then**
- 20: $\rho \leftarrow \frac{\rho}{2}$
- 21: end if

22: end while

Algorithm 6 EnNSGAII algorithm in the form of Pseudo-code.

- Inputs: L/U: lower/upper bound; N_{pop}: number of bids; N_{var}: number of variables; NFE_{max}: maximum number of function evaluations; prob: the probability of a crossover, η: the distribution index.
- 2: Initialise population of N_{pop} candidate solution using the representation introduced in Section 6.1.
- 3: Calculate objective function values of all candidate solutions (Section 6.2).
- 4: Assign (level) rank based on Pareto sorting.
- 5: $NFE = N_{pop}$
- 6: *iter* = 0
- 7: while $NFE <= NFE_{max}$ do
- 8: iter = iter + 1
- 9: Perform Tournament selection (Section 6.3.1).
- 10: Perform SBX mutation (Section 6.3.1).
- 11: Perform Polynomial mutation (Section 6.3.1).
- 12: Calculate objective function values of all new candidate solutions (Section 6.2).
- 13: $R_t \leftarrow \text{Combine parent and offspring population.}$
- 14: Assign (level) rank based on Pareto sorting.
- 15: Generate sets of non-dominated solutions.
- 16: Add solutions to next generation starting from the first front to N_{pop} individuals.
- 17: Determine crowding distance.
- 18: Select points on the lower front with high crowding distance.
- 19: Update NEF.
- 20: end while

complexity is $O(N^2)$. The quantisation stage involves dividing the DCT coefficients by quantisation values and rounding to the

nearest integer, and its computational complexity is O(N). The entropy coding stage here was based on the Huffman coding, with a computational complexity of O(NlogN). Therefore, the overall computational complexity of JPEG image compression is $O(N^2)$.

EnMOGA algorithm: Our proposed EnMOGA algorithm includes four main operators, tournament selection, SBX mutation operator, objective function, and Polynomial mutation. Other parts of EnMOGA are computationally inexpensive. The computational complexity of tournament selection is $O(mN_{non})$, where *m* is the number of candidate solutions in the tournament. This is because in tournament selection, m candidate solutions are randomly selected from the population, and the best individual among these *m* candidate solutions is selected for reproduction. This process is repeated for each candidate solution in the new population. In SBX, each dimension of the offspring is generated by combining the corresponding dimensions of the two parent solutions. The generation of each dimension is a computationally inexpensive operation, which contributes to the overall O(d) time complexity of SBX. In addition, in Polynomial mutation, each dimension of the solution is mutated by applying a polynomial function. The computation of the polynomial function is the main operation in Polynomial mutation, and it is a computationally inexpensive operation, contributing to the overall O(d) time complexity. Therefore, the complexity of one iteration in total is $O(mN_{pop} + d + d + N^2)$. In the real world, the number of pixels in a typical image is usually bigger than the population size. For instance, the population size is usually selected as a number between 5 and 200, while the number of pixels for a small image with a size of 75 \times 75 is 5625. Also, *m* is a small number less than the population size. Therefore, assuming that the number of pixels is significantly larger than the population size, we can conclude that the computational complexity of the EnMOGA algorithm for one iteration is $O(N_{pop}N^2)$, and for the whole algorithm is $O(IN_{pop}N^2)$.

EnMODE algorithm: Our proposed EnMODE consists of three computational components of Dither, standard mutation, and crossover. The computational complexity of the Dither operation is O(d) since it only involves simple arithmetic operations on *d*-dimensional vectors. The computational complexity for the mutation and crossover is also O(d). Also, in each iteration, DE requires to evaluate N_{pop} candidate solutions. Since the computation complexity of the objective function is significantly larger than O(d), the computational complexity of the EnMODE algorithm is also $O(IN_{pop}N^2)$.

EnMOPSO algorithm: in this paper, we used an adaptive PSO, in which the computational components are position updating strategy, estimation of evolutionary states, ω updating, and objective function. In each iteration of PSO, the position of each particle is updated based on its current velocity and a combination of its personal best solution and the global best solution found so far. This update involves performing d dimensional vector arithmetic operations, including addition, subtraction, and multiplication. These operations are relatively simple and computationally efficient. Therefore, the computational complexity of the updating strategy is O(d). The computation complexity of the evolutionary states estimation process is $O(N_{pop}d)$. Also, ω updating benefits from addition, division, and multiplication, resulting in O(1). Since again $O(N^2)$ is significantly bigger than $O(N_{pop}d)$, the computational complexity of EnMOPSO is also $O(IN_{pop}N^{\frac{1}{2}}).$

EnMOES algorithm: For the EnMOES algorithm, we have the same conditions as the earlier algorithms and the most dominant component based on the computational complexity is the objective function. Therefore, again EnMOES algorithms benefit from a computational complexity of $O(IN_{pop}N^2)$.

EnMOPS algorithm: in each iteration of EnMOPS, two equations should be calculated that are computationally inexpensive since they only include addition and subtraction operations. Also, the objective function should be computed 2 times in each iteration. As a result, since EnMOPS is a single-solution-based MA, the computation complexity of EnMOPS is $O(IN^2)$.

EnNSGAII algorithm: from [19], the computational complexity of NSGA-II for each iteration is $O(MN_{pop}^2)$, where *M* is the number of objective functions. In the paper, the authors assumed that the objective function is computationally inexpensive. Therefore, for this problem, we should add the objective function to the computation complexity. In addition, the computational complexity of the operators, as already mentioned, is significantly lower than $O(N^2)$. In other words, it can be seen as $O(I(N_{pop}^2 + N_{pop}N^2))$. With our previous assumption on the number of pixels and population size, the overall complexity of EnNSGAII is $O(IN_{pop}N^2)$, which is governed by the objective function.

EnNSGAIII algorithm: [23] indicates that the computational complexity of NSGA-III for one generation is $O(MN_{pop}^2)$. Therefore, We have the same circumstance with EnNSGAII for the EnNSGAIII algorithm, resulting in a computational complexity of $O(IN_{pop}N^2)$.

All in all, the overall computational complexity of all algorithms, except one, is the same, which is governed by the population size and the objective function. It is worthwhile to note that, to have a fair comparison in the experimental section, the stopping criterion is selected as the number of function evaluations rather than the number of iterations, leading to granting them the same computational budget.

6.6. Limitations

Despite the effectiveness of the proposed approach, it suffers from several limitations. The first challenge is finding priorities. In other words, what is a higher-priority goal for a developer? Sometimes even a developer does not know which objective has higher priority. Therefore, he/she will not be able to give proper weights to the objective functions. Despite the existence of this challenge, it can be moderated by using default weights for each objective in case a developer does not have an idea in mind.

Another limitation is to ignore the user's opinion regarding the image properties. As mentioned, in multi-objective optimisation, there are several objectives. The importance of each objective function is determined using weights in a priori algorithms. In other words, a user can embed the importance of the objectives as the preference in the objective function. For example, if the file size is more important to the user compared to the quality, more weight is given to the file size in the objective function. Despite embedding this preference, the output may not be what the user intended since only the importance of the functions is considered in terms of each other. Assume an Android developer is thinking about creating a small-sized app. Here, the user gives more weight to the file size than the quality, but the exact amount of size the user is considering is not stated. With this app's optimisation algorithms, the algorithm might find an image with a low file size, but it might be very different from the desired file size. In other words, the exact user's opinion about the specification of the output file is not considered in the proposed approach.

7. Experimental results

To demonstrate the superiority of our proposed strategy, an extensive set of experiments is provided. To this end, we have used 7 popular benchmark images in image compression, including, *Airplane*, *Barbara*, *Lena*, *Mandrill*, *Peppers*, *Tiffany*, and *Sailboat*, as well as 6 images suggested in [52] for image quantisation

Table 3 Parameter settings.		
Algorithm	Parameter	Value
	Prob for crossover	0.9
EpMOCA	η for crossover	20
EIIWOGA	Prob for mutation	0.3
	η for mutation	20
EnMOPSO	-	-
EnMODE	CR	0.2
EnMOES	-	-
EnMOPS	ρ	0.5
	Prob for crossover	0.9
EDNSCA II	η for crossover	20
EIINSGA-II	Prob for mutation	0.3
	η for mutation	20
	Prob for crossover	0.9
E-NECA III	η for crossover	20
EIINSGA-III	Prob for mutation	0.3
	η for mutation	20

Algorithm 7 EnNSGAIII algorithm in the form of Pseudo-code.

- 1: **Inputs:** L/U: lower/upper bound; N_{pop} : number of bids; N_{var} : number of variables; NFE_{max} : maximum number of function evaluations; *prob*: the probability of a crossover, η : the distribution index.
- 2: Initialise population of N_{pop} candidate solution using the representation introduced in Section 6.1.
- 3: Calculate objective function values of all candidate solutions (Section 6.2).
- 4: Assign (level) rank based on Pareto sorting.
- 5: $NFE = N_{pop}$
- 6: *iter* = 0^{1}
- 7: while $NFE <= NFE_{max}$ do
- 8: iter = iter + 1
- 9: Perform Tournament selection (Section 6.3.1).
- 10: Perform SBX mutation (Section 6.3.1).
- 11: Perform Polynomial mutation (Section 6.3.1).
- 12: Calculate objective function values of all new candidate solutions (Section 6.2).
- 13: $R_t \leftarrow$ Combine parent and offspring population.
- 14: Assign (level) rank based on Pareto sorting.
- 15: Generate sets of non-dominated solutions.
- 16: Add solutions to next generation starting from the first front to N_{pop} individuals.
- 17: Normalise objective function and create reference set.
- 18: Assign each member to a reference point.
- 19: Compute niche count of each reference point.
- 20: Add new members to the new population based on the niche count.
- 21: Update NEF.
- 22: end while

benchmarking, including, *Snowman*, *Beach*, *Cathedrals beach*, *Dessert*, *Headbands*, and *Landscape*. Fig. 8 shows the benchmark images.

Our proposed strategy is embedded in five scalarisation and two Pareto-based methods. All algorithms are run 30 times independently to provide a fair comparison, and their statistical results, including average and standard deviation, are presented. The population size and the number of function evaluations for all algorithms are set to 50 and 1000, respectively. For other parameters, we used the default parameters that can be seen in Table 3. All algorithms are implemented in Python and with the Pymoo framework [53], an open-source framework including state-of-the-art single-and multi-objective algorithms as well as features related to multi-objective optimisation such as visualisation, introduced in 2020.

7.1. Results of scalarisation approaches

This section aims to find answers to two central questions, as follows:

- Is the proposed strategy able to provide higher quality results than the baseline algorithm? (Here, the baseline algorithm means the standard JPEG compression method).
- Among the embedded scalarisation methods, which algorithm has been able to provide a better solution?

To this end, Table 4 compares the results of the baseline algorithm with other algorithms in terms of mean and standard deviation of the objective function yielded by each algorithm. All algorithms except the baseline algorithm are run 30 times since the baseline algorithm is deterministic. Therefore, we do not provide any standard deviation for the baseline algorithm. Also, the rank of each algorithm per image is indicated from the smallest mean to the highest mean in Table 4. In the last row of the table, the average rank of each algorithm and subsequently, the overall ranks are reported as well.

From Table 4, and by a comparison between the baseline algorithm with others, we can observe that the baseline algorithm in all cases and in comparison to all algorithms achieves the worst results. For instance, for the *Airplane* image, the objective function for the baseline algorithm is 1.7839, while for others, it is between 1.4185 and 1.4700, indicating a significant improvement in the proposed strategy. Therefore, in short, we can say that our strategy, independent of the embedding algorithm, can provide competitive results compared to the baseline algorithm.

As mentioned, we employed five scalarisation methods. Here, we compare the results of scalarisation methods together. The results can be seen in Table 4. From the table, we can observe that EnMOGA can achieve the first rank in 6 out of 13 images and the second rank in 7 out of 13 images. Also, EnMOPS is placed in the first rank with seven cases, and in the second rank with five images. EnMOES achieved the fifth or worst rank among the embedding algorithms in all cases. Therefore, from the last row of the table, we can say that EnMOGA and EnMOPS provide the



(a) Airplane





(c) Lena







(e) Peppers



(f) Sailboat



(g) Snowman



(h) Tiffany



(i) Beach



(j) Cathedrals beach



(k) dessert



(l) headbands



(m) landscape

Fig. 8. Benchmark images.

Table 4

A comparison between different scalarisation approaches and the baseline algorithm in terms of objective function.

Images		Baseline	EnMOGA	EnMOPSO	EnMODE	EnMOES	EnMOPS
Airplane	Mean	1.7839	1.4227	1.4370	1.4596	1.4700	1.4185
	Std.	-	0.0055	0.0064	0.0051	0.0034	0.0159
	rank	6	2	3	4	5	I
Barbara	Mean	1.7647	1.3571	1.3688	1.4052	1.4224	1.3731
	Std.	-	0.0076	0.0079	0.0079	0.0060	0.0265
	rank	6	1	2	4	5	3
Lena	Mean	1.8552	1.4312	1.4475	1.4665	1.4794	1.4337
	Std.	-	0.0062	0.0072	0.0054	0.0029	0.0205
	rank	6	1	3	4	5	2
Mandrill	Mean	1.8961	1.6098	1.6329	1.6666	1.6767	1.5703
	Std.	-	0.0073	0.0084	0.0041	0.0049	0.0196
	rank	6	2	3	4	5	1
Peppers	Mean	1.8604	1.5168	1.5313	1.5409	1.5507	1.5004
	Std.	-	0.0046	0.0048	0.0044	0.0036	0.0120
	rank	6	2	3	4	5	1
Sailboat	Mean	1.8458	1.5670	1.5839	1.6015	1.6103	1.5379
	Std.	-	0.0048	0.0063	0.0033	0.0027	0.0150
	rank	6	2	3	4	5	1
Snowman	Mean	1.8239	1.4041	1.4150	1.4467	1.4611	1.4120
	Std.	-	0.0071	0.0075	0.0080	0.0054	0.0189
	rank	6	1	3	4	5	2
Tiffany	Mean	1.8027	1.5109	1.5147	1.5229	1.5282	1.4867
•	Std.	-	0.0018	0.0024	0.0026	0.0023	0.0093
	rank	6	2	3	4	5	1
Beach	Mean	1.8047	1.4847	1.4956	1.5282	1.5386	1.4723
	Std.	-	0.0061	0.0081	0.0058	0.0043	0.0186
	rank	6	2	3	4	5	1
Cathedrals beach	Mean	1.7551	1.3648	1.3748	1.4013	1.4110	1.3647
	Std.	-	0.0076	0.0080	0.0056	0.0056	0.0175
	rank	6	2	3	4	5	1
Dessert	Mean	1.8195	1.3760	1.3859	1.4113	1.4220	1.3804
	Std.	-	0.0067	0.0066	0.0076	0.0053	0.0180
	rank	6	1	3	4	5	2
Headbands	Mean	1.8027	1.3879	1.3971	1.4201	1.4269	1.3901
	Std.	-	0.0047	0.0058	0.0042	0.0037	0.0141
	rank	6	1	3	4	5	2
Landscape	Mean	1.8456	1.4272	1.4386	1.4731	1.4940	1.4303
-	Std.	-	0.0066	0.0081	0.0087	0.0058	0.0203
	rank	6	1	3	4	5	2
Average rank		6	1.54	2.92	4.00	5.00	1.54
Overall rank		6	1.5	3	4	5	1.5
		-		-		-	

best average rank and, subsequently, overall rank, while EnMOES provides the highest average and overall ranks.

Due to the non-deterministic behaviour of metaheuristic algorithms, non-parametric statistical is obligatory. In this case, the alternative hypothesis H₁ denotes a statistically significant difference between the algorithms, while the null hypothesis H_0 states that there is no statistical difference between the two algorithms. The null hypothesis is the initial statistical assertion, and the alternative hypothesis would be accepted if the null hypothesis were to be shown to be false. To this end, we carried out Wilcoxon signed rank test [54] at 5% significance level based on the mean objective function value to compare the results statistically. We selected the Wilcoxon signed rank test since it does not assume normal distributions and therefore, it is safer than the t-test. Also, the outliers (exceptionally good/bad performances of a few problems) have less impact on the Wilcoxon test in comparison to the t-test [54]. Because of these reasons, in [54], Wilcoxon signed rank test is introduced as a proper alternative to assess the MA algorithms statistically.

Table 5 shows that EnMOGA and EnMOPS perform statistically superior to other algorithms since both win in 4 cases. Also, EnMOPS and EnMOGA are statistically the same. The overall following best working algorithm is EnMOPSO (3 wins, 2 losses). Among the scalarisation algorithms, EnMOES performs worst (1 win, 4 losses). Again, the baseline algorithm (0 win, 5 losses) fails against all proposed scalarisation methods.

Comparing between EnMOPS and others, we can realise that the number of iterations for EnMOPS is higher than others since the stopping criterion is the number of objective function values and EnMOPS lacks a population. In other words, there is a lower number of iterations and a larger population size for populationbased algorithms. Therefore, we can say that in this problem, more evolution of a candidate solution is more effective than distributing the computations through the evolution of several candidate solutions. Also, by comparison between EnMOES (as one of the worst algorithms) and EnMOPS algorithms (as one of the best-performing algorithms), we can observe two leading differences: 1) EnMOPS is a single-solution-based algorithm, while EnMOES is a population-based algorithm and 2) EnMOPS is based on step size, meaning that the step value is decreasing over time, whereas EnMOPS only employs a normal distribution for the updating process. As a result, another reason for the remarkable efficiency of EnMOPS might be due to the decreasing step size.

We also investigated the convergence curves for all algorithms. Fig. 9 shows plots of objective function values against the number of function evaluations on all images and for a single

Table 5

Results of Wilcoxon signed rank test based on the mean objective function value. \ddagger, \dagger , and \approx indicate that the algorithm in the corresponding row is statistically better than, worse than, or similar to the algorithm in the corresponding column. The last column summarises the algorithms' total wins (w), ties (t), and losses (l).

	Baseline	EnMOGA	EnMOPSO	EnMODE	EnMOES	EnMOPS	w/t/l
Baseline		t	t	t	†	†	0/0/5
EnMOGA	‡		‡	‡	‡	\approx	4/1/0
EnMOPSO	‡	t		‡	‡	†	3/0/2
EnMODE	1	t	t		‡	-	2/0/3
EnMOES	‡	†	†	†		†	1/0/4
EnMOPS	‡	\approx	‡	ţ	‡		4/1/0

random run. It is worthwhile to mention that four scalarisation algorithms, EnMOGA, EnMODE, EnMOPSO, EnMOES are populationbased algorithms, while EnMOPS is a single-solution algorithm. In other words, EnMOPS starts with only one solution, unlike other algorithms that start with a population of solutions. Also, the *x*axis curve in the convergence curves is based on the number of function evaluations (and not the number of iterations). As a result, the start point of EnMOPS is different from others in the convergence curves. It can be seen that EnMOPS and EnMOGA have faster convergence compared to other algorithms, while EnMOES suffers from low-speed convergence.

7.2. Results of pareto-based approaches

The evaluation method of the Pareto-based approaches differs from the scalarisation approaches because the Pareto-based approaches result in several solutions while the scalarisation methods produce only one solution. There are several measures to validate the results of Pareto-based approaches, such as Generational Distance (GD) [55] and Inverted Generational Distance (IGD) [56], while here, we cannot use these because they require the true Pareto front, which is not available in this problem. Therefore, we used an alternative measure, called hyper-volume (HV) [57], which does not require a true Pareto-front to validate the results. The HV measure is regarded as a fair measure among other criteria [58] so that HV can take into consideration both closeness to the optimal solution and being welldistributed along the whole Pareto front. HV measure determines the area/volume that, in relation to a reference point, is dominated by the given set of solutions. A higher value of HV measure in a minimisation problem shows a better quality of the solution.

This paper integrated the proposed strategy into two Paretobased algorithms, including NSGA-II and NSGA-III. The results based on the HV measure are given in Table 6. The table shows that EnNSGAII outperforms EnNSGAIII in 10 out of 13 cases, while it fails in 3 cases. From the last row of the table, we can observe that the average rank of EnNSGAII is lower than EnNSGAIII. In other words, EnNSGAII overcomes EnNSGAIII.

To perform a deeper analysis, we also conducted a Wilcoxon signed rank test on the results. The achieved *p*-value is 0.0574, which means that there is a statistical difference between the two algorithms only at a 10% significance level. This was expected because in most cases, reference points showed better performance against crowding distance when the number of objectives is greater than 2 [23].

Finally, to have a more comprehensive view of the generated Pareto fronts, we plot the Pareto front for the algorithms in Fig. 11. It can be seen that EnNSGAII provides more points in the Pareto front compared to EnNSGAIII.

Since there are two conflicting objective functions, we cannot plot convergence curves in terms of objective functions. Therefore, in the next experiment, we indicated the convergence curves in terms of the HV measure rather than objective functions. Fig. 10 shows the convergence curves for our two proposed

Table 6

A	comparison	between	EnNSGAII	and	EnNSGAIII	in	terms	ot	the	ΗV	measure.
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Images		EnNSGAII	EnNSGAIII
Airplane	Mean	7.0803	7.0920
	Std.	0.1805	0.2133
	rank	2	1
Barbara	Mean	7.1164	7.0369
	Std.	0.1717	0.2482
	rank	1	2
Lena	Mean	7.1206	7.1436
	Std.	0.2085	0.1077
	rank	2	1
Mandrill	Mean	7.0618	7.0585
	Std.	0.0483	0.0646
	rank	1	2
Peppers	Mean	7.2614	6.6218
	Std.	0.0634	0.5518
	rank	1	2
Sailboat	Mean	7.0653	7.0424
	Std.	0.1971	0.0986
	rank	1	2
Snowman	Mean	7.1900	7.0897
	Std.	0.0599	0.0727
	rank	1	2
Tiffany	Mean	6.9587	7.1036
	Std.	0.4292	0.1055
	rank	2	1
Beach	Mean	7.078745	6.960397
	Std.	0.03692	0.1994
	rank	1	2
Cathedrals beach	Mean	7.1510	7.1149
	Std.	0.1207	0.0571
	rank	1	2
Dessert	Mean	7.1242	7.0103
	Std.	0.2396	0.3402
	rank	1	2
Heatbands	Mean	7.2309	7.1754
	Std.	0.0521	0.1084
	rank	1	2
Landscape	Mean	7.1799	7.1417
	Std.	0.0765	0.0837
	rank	1	2
Average rank		1.23	1.77
Overall rank		1	2

algorithms. It is clear that, in most cases, EnNSGAII provides a faster convergence rate.

7.3. Comparison between scalarisation and Pareto-based methods

Generally speaking, a comparison between scalarisation and Pareto-based methods is not possible since scalarisation methods only generate one solution based on a given set of weights, while Pareto-based methods generate a set of solutions. To tackle this



Fig. 9. Convergence curves for the scalarisation approaches.

problem, we select one solution from a Pareto-based method as

$$PF_{selected} = \min_{i=1}^{N_{PF}} (w_1 f_1^i(x) + w_2 f_2^i(x) + \dots + w_M f_M^i(x))$$
(29)

where N_{PF} is the number of solutions in the generated Pareto front, w_1, \ldots, w_M are the corresponding weights in the scalarisation method, and f_1^i, \ldots, f_M^i are the objective function values for the *i*th solution in the Pareto front.



Fig. 10. Convergence curves for the Pareto-based approaches in terms of the HV measure.

Table 7 shows the results. It can be seen that EnNSGAII cannot work better than two others when we select only one solution, as expected since the Pareto-based approaches focus on a set of solutions and not only one solution. Despite the performance of the scalarisation method compared to the Pareto-based algorithm, we cannot say that the Pareto-based algorithm did not work well because the output of the Pareto-based algorithm is a set of solutions with different weights, while the scalarisation method does not have such an ability. Also, from this experiment, it is worthwhile to mention that if we know the weights of each objective, scalarisation methods are preferable.

7.4. Sensitivity analysis

The sensitivity analysis of the suggested algorithm's control parameters is examined below. To serve as representatives, we



Fig. 11. A comparison of the Pareto front of the proposed methods for each image.

chose two images, namely *Airplane* and *Barbara*. We selected En-MOGA as a scalarisation method, and EnNSGAII as a Pareto-based method, which show better performance, for our experiments.

7.4.1. Sensitivity to population size

Population size is one of the most critical parameters in metaheuristic algorithms. Metaheuristic algorithms with a large population size usually provide better results than small population size since a large population size supports higher diversity for the population, leading to higher exploration ability due to the recombination of its diverse members [59,60]. Nevertheless, sometimes it is more effective to use a small population size. The term micro-algorithm, μ -algorithm, refers to a metaheuristic algorithm with a small population size [60].

This section aims to investigate the effect of population size on performance. To this end, the population size is set to 5, 10, 20, 30, 50, 100, and 200, while the number of function evaluations is fixed for all algorithms. In other words, for smaller population





Fig. 12. The objective function values obtained with different population sizes.

Table 7

A comparison between the scalarisation and Pareto-based methods. The values signify the objective function value defined for scalarisation method and $PF_{selected}$ for EnNSGAII. The best result for a given image is boldfaced.

Images	EnMOGA	EnMOPS	EnNSGAII
Airplane	1.4227	1.4185	1.4705
Barbara	1.3571	1.3731	1.4048
Lena	1.4312	1.4337	1.4720
Mandrill	1.6098	1.5703	1.6695
Peppers	1.5168	1.5004	1.5462
Sailboat	1.5670	1.5379	1.6076
Snowman	1.4041	1.4120	1.5212
Tiffany	1.5109	1.4867	1.4207
Beach	1.4847	1.4723	1.3987
Cathedrals beach	1.3648	1.3647	1.4155
Dessert	1.3760	1.3804	1.4459
Headbands	1.3879	1.3901	1.5244
Landscape	1.4272	1.4303	1.4648

size, the number of iterations is higher than for larger population size. Fig. 12(a) shows the objective function value achieved by different population sizes and for the EnMOGA algorithm. Both

images show an upward trend; in other words, a larger population size leads to a higher objective function value. It means that lower population size is preferable.

The same experiment is performed by EnNSGAII, and the results are given in Fig. 12(b). For the *Airplane* image, by increasing the population size from 5 to 20, the HV value is also increased, while there is a downward trend by increasing the population size from 20 to 200. For the *Barbara* image, the conditions are a bit different, and the trend is downward for all population sizes.

In short, it can be said that smaller population sizes can lead to better results for the scalarisation approach, whereas the Paretobased approach works better with larger population sizes (see Fig. 12).

7.4.2. Sensitivity to prob and η in the crossover operator

Our crossover operator depends on two parameters, called *prob* and η . To study the sensitivity of *prob* and η , 18 combinations of *prob* and η are assessed (*prob* = 0.5, 0.7, 0.9 and η = 2, 5, 10, 20, 30, 40). All other parameters are fixed. Figs. 13 indicates the objective function value with different *prob* and η combinations for the *Airplane* and *Barbara* images. For the





(b) Barbara image

Fig. 13. The objective function values of different prob and η values in the crossover operator of EnMOGA.

Airplane image, we can see that the objective function value for prob = 0.5 is higher than the other two for all η values. There is a fluctuation in comparison between prob = 0.7 and prob = 0.9; meaning that for $\eta = 2, 5, 20, prob = 0.7$ outperforms prob = 0.9, while for other η values, prob = 0.9 provides better results than prob = 0.7.

This experiment is also conducted for the EnNSGAII algorithm. The results are given in Fig. 14. From Fig. 14(a), it is clear that prob = 0.9 provides better results with more stability in all cases. By increasing the η values for prob = 0.5 and 0.7, the HV values also are improved. The similar results can be seen in Fig. 14(b) for the *Barbara* image. In most cases, the prob = 0.9 outperforms other *prob* values, followed by prob = 0.7.

7.4.3. Sensitivity to prob and η in the mutation operator

There are also two parameters, *prob* and η , in the mutation operator. We investigated the effect of 18 combinations (*prob* = 0.1, 0.3, 0.5 and $\eta = 2, 5, 10, 20, 30, 40$). The results of EnMOGA are given in Fig. 15. For the *Airplane* image and *prob* = 0.5, η values have a downward trend; in other words, by increasing the η values, performance is also improved. For $\eta = 0.3$, there is a fluctuation, while for $\eta = 0.1$, the results are more stable. From Fig. 15(b), we can observe that the EnMOGA is sensitive to these parameters. In particular, $\eta = 20$ and *prob* = 0.3 provided the

best results, whereas the worst results are achieved by $\eta = 40$ and prob = 0.5.

Similar results for EnNSGAII in Fig. 16 indicate that, again, these parameters can affect the performance. Fig. 16(a) investigates that $\eta = 20$ can provide the highest HV values for most cases. Also, there is an upward trend from *eta* = 2 to *eta* = 20, while a downward trend can be seen from *eta* = 20 to *eta* = 40. The similar trends can also be observed in Fig. 16(b) for Barbara image.

8. Conclusions

This paper proposes an energy-aware multi-objective strategy to find the best values of quantisation tables in JPEG image compression. To this end, first, we investigated whether there is a high correlation between two main properties of images, image quality and file size, and energy consumption. As a result, these two can be considered a proxy for energy consumption. Then, we defined two conflicting objective functions, including image quality and file size, while a vector-based representation was used as the candidate solution. In the next step, we embedded the proposed strategy into seven metaheuristic algorithms. Five of them are among scalarisation methods, including energy-aware multi-objective genetic algorithm (EnMOGA),



(a) Airplane image



Fig. 14. The HV values of different *prob* and η values in the crossover operator of EnNSGAII.

energy-aware multi-objective particle swarm optimisation (EnMOPSO), energy-aware multi-objective differential evolution (EnMODE), energy-aware multi-objective evolutionary strategy (EnMOES), and energy-aware multi-objective pattern search (En-MOPS), while two others are selected among Pareto-based approaches, including energy-aware non-dominated sorting genetic algorithm (EnNSGA-II) and energy-aware reference-based NSGA-II (EnNSGA-III). Our extensive results indicated that all algorithms could outperform the baseline. In particular, EnMOGA, EnMOPS, and EnNSGA-II offered better results.

Despite the effectiveness of the proposed strategy, this work has two main limitations. First, our proposed approach takes the importance of each objective function into consideration as the preference, while it ignores the user's opinion regarding the output image properties. For example, the goal might be to have a small size image, while this method may provide a larger size image. Another limitation is in the selection of priorities, given that sometimes even a developer does not have enough knowledge to choose a priority.

This work can be extended in the future with the following hints.

- This paper employed some well-established metaheuristic algorithms for the embedding process, while it can be improved by embedding the strategy into more recent algorithms such as L-SHADE [61].
- This paper ignored decomposition-based approaches (DBA) for finding the conflicting objectives, while the literature shows that DBAs have an excellent capability for multiobjective optimisation. Therefore, DBAs can be used for this problem in the future.
- The goal of this paper was to find the optimal points for quantisation tables. It will likely to provide better results by adding other parameters to the current representation, such as the quality factor.
- This paper employed the default parameter of algorithms for the first stage of the embedding process, and we have not focused on the parameter settings for all algorithms. The optimal parameters also can be archived by a self-adaptation approach.
- This paper only employed two objective functions, while this research can be extended to the many-objective optimisation problem in the quantisation table generation.



(a) Airplane image





Fig. 15. The objective function values of *prob* and η in the mutation operator of EnMOGA.



(a) Airplane image



(b) Barbara image

Fig. 16. The HV values of *prob* and η in the mutation operator of EnNSGAII.

• This paper does not into consideration the user's opinion regarding the output image properties. The authors intend to propose a user-specified JPEG image compression in the future.

CRediT authorship contribution statement

Seyed Jalaleddin Mousavirad: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing – original draft, Visualization, Writing – review & editing. **Luís A. Alexandre:** Conceptualization, Methodology, Validation, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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References

- Hammad Khalid, Emad Shihab, Meiyappan Nagappan, Ahmed E. Hassan, What do mobile app users complain about? IEEE Softw. 32 (3) (2014) 70–77.
- [2] Jagannath Singh, Arpan Maity, Energy consumption-based profiling of Android apps, in: Mobile Application Development: Practice and Experience: 12th Industry Symposium in Conjunction with 18th ICDCIT 2022, Springer, 2023, pp. 21–32.
- [3] Geoffrey Hecht, Naouel Moha, Romain Rouvoy, An empirical study of the performance impacts of Android code smells, in: Proceedings of the International Conference on Mobile Software Engineering and Systems, 2016, pp. 59–69.
- [4] Sona Mundody, K. Sudarshan, Evaluating the impact of Android best practices on energy consumption, in: IJCA Proceedings on International Conference on Information and Communication Technologies, Vol. 8, 2014, pp. 1–4.
- [5] Emanuele Iannone, Manuel De Stefano, Fabiano Pecorelli, Andrea De Lucia, Predicting the energy consumption level of Java classes in Android apps: An exploratory analysis, in: Proceedings of the 9th IEEE/ACM International Conference on Mobile Software Engineering and Systems, 2022, pp. 1–5.
- [6] Abdul Ali Bangash, Karim Ali, Abram Hindle, A black box technique to reduce energy consumption of Android apps, in: Proceedings of the ACM/IEEE 44th International Conference on Software Engineering: New Ideas and Emerging Results, 2022, pp. 1–5.
- [7] Leonhard Wattenbach, Basel Aslan, Matteo Maria Fiore, Henley Ding, Roberto Verdecchia, Ivano Malavolta, Do you have the energy for this meeting? An empirical study on the energy consumption of the google meet and zoom Android apps, in: Proceedings of the 9th IEEE/ACM International Conference on Mobile Software Engineering and Systems, 2022, pp. 6–16.
- [8] A. Andreadis, G. Benelli, A. Garzelli, S. Susini, A DCT-Based adaptive compression algorithm customized for radar imagery, in: IGARSS'97. 1997 IEEE International Geoscience and Remote Sensing Symposium Proceedings. Remote Sensing-A Scientific Vision for Sustainable Development, Vol. 4, IEEE, 1997, pp. 1993–1995.
- [9] Jinyoung Choi, Bohyung Han, Task-aware quantization network for JPEG image compression, in: European Conference on Computer Vision, Springer, 2020, pp. 309–324.

- [10] O. Ferrer-Roca, R.J. Rodriguez, A. Sousa Pereira, Annex x: Image formats, in: Handbook of Telemedicine, IOS Press, 1998, pp. 252–261.
- [11] Darrell Whitley, A genetic algorithm tutorial, Stat. Comput. 4 (2) (1994) 65–85.
- [12] Yuhui Shi, Russell Eberhart, A modified particle swarm optimizer, in: IEEE International Conference on Evolutionary Computation, 1998, pp. 69–73.
- [13] Balasubramanian Vinoth Kumar, Manavalan Karpagam, Differential evolution versus genetic algorithm in optimising the quantisation table for JPEG baseline algorithm, Int. J. Adv. Intell. Paradigms 7 (2) (2015) 111–135.
- [14] Milan Tuba, Nebojsa Bacanin, JPEG quantization tables selection by the firefly algorithm, in: 2014 International Conference on Multimedia Computing and Systems, ICMCS, IEEE, 2014, pp. 153–158.
- [15] Leonardo Faria Costa, Antônio Cláudio Paschoarelli Veiga, Identification of the best quantization table using genetic algorithms, in: PACRIM. 2005 IEEE Pacific Rim Conference on Communications, Computers and Signal Processing, 2005, IEEE, 2005, pp. 570–573.
- [16] Qijun Wang, Ping Liu, Lei Zhang, Fan Cheng, Jianfeng Qiu, Xingyi Zhang, Rate-distortion optimal evolutionary algorithm for JPEG quantization with multiple rates, Knowl.-Based Syst. 244 (2022) 108500.
- [17] Dario Di Nucci, Fabio Palomba, Antonio Prota, Annibale Panichella, Andy Zaidman, Andrea De Lucia, Petra: A software-based tool for estimating the energy profile of Android applications, in: 2017 IEEE/ACM 39th International Conference on Software Engineering Companion, ICSE-C, IEEE, 2017, pp. 3–6.
- [18] Nyoman Gunantara, A review of multi-objective optimization: Methods and its applications, Cogent Eng. 5 (1) (2018) 1502242.
- [19] Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal, T.A.M.T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Trans. Evol. Comput. 6 (2) (2002) 182–197.
- [20] Fawad Ahmed, Shunmin Zhu, Guoyao Yu, Ercang Luo, A potent numerical model coupled with multi-objective NSGA-II algorithm for the optimal design of stirling engine, Energy 247 (2022) 123468.
- [21] Zoheir Babajamali, Farshid Aghadavoudi, Fatemeh Farhatnia, S. Ali Eftekhari, Davood Toghraie, et al., Pareto multi-objective optimization of tandem cold rolling settings for reductions and inter stand tensions using NSGA-II, ISA Trans. 130 (2022) 399–408.
- [22] Yu Xue, Haokai Zhu, Ferrante Neri, A feature selection approach based on NSGA-II with ReliefF, Appl. Soft Comput. (2023) 109987.
- [23] Kalyanmoy Deb, Himanshu Jain, An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints, IEEE Trans. Evol. Comput. 18 (4) (2013) 577–601.
- [24] Farzad Mostafazadeh, Saeed Jalilzadeh Eirdmousa, Mehdi Tavakolan, Energy, economic and comfort optimization of building retrofits considering climate change: A simulation-based NSGA-III approach, Energy Build. 280 (2023) 112721.
- [25] Run Chen, Determination of fatty acid of wheat by near-infrared spectroscopy with combined feature selection based on CARS and NSGA-III, Infrared Phys. Technol. (2023) 104572.
- [26] Beatrice Lazzerini, Francesco Marcelloni, Massimo Vecchio, A multiobjective evolutionary approach to image quality/compression trade-off in JPEG baseline algorithm, Appl. Soft Comput. 10 (2) (2010) 548–561.
- [27] Mario Konrad, Herbert Stogner, Andreas Uhl, Evolutionary optimization of JPEG quantization tables for compressing Iris polar images in Iris recognition systems, in: 2009 Proceedings of 6th International Symposium on Image and Signal Processing and Analysis, IEEE, 2009, pp. 534–539.
- [28] Yung-Gi Wu, Ga-based DCT quantization design for medical images, in: Sixth International Conference on Quality Control by Artificial Vision, Vol. 5132, SPIE, 2003, pp. 7–16.
- [29] B. Vinoth Kumar, G.R. Karpagam, S.P. Naresh, Generation of JPEG quantization table using real coded quantum genetic algorithm, in: 2016 International Conference on Communication and Signal Processing, ICCSP, IEEE, 2016, pp. 1705–1709.
- [30] Vinoth Kumar Balasubramanian, Karpagam Manavalan, Knowledge-based genetic algorithm approach to quantization table generation for the JPEG baseline algorithm, Turk. J. Electr. Eng. Comput. Sci. 24 (3) (2016) 1615–1635.
- [31] Rainer Storn, Kenneth Price, Differential evolution–A simple and efficient heuristic for global optimization over continuous spaces, J. Global Optim. 11 (4) (1997) 341–359.
- [32] B. Vinoth Kumar, G.R. Karpagam, Knowledge-based differential evolution approach to quantisation table generation for the JPEG baseline algorithm, Int. J. Adv. Intell. Paradigms 8 (1) (2016) 20–41.
- [33] B. Vinoth Kumar, G.R. Karpagam, Reduction of computation time in differential evolution-based quantisation table optimisation for the JPEG baseline algorithm, Int. J. Comput. Syst. Eng. 4 (1) (2018) 58–65.
- [34] Xin-She Yang, Firefly algorithm, stochastic test functions and design optimisation, 2010, arXiv preprint arXiv:1003.1409.
- [35] Eva Tuba, Milan Tuba, Dana Simian, Raka Jovanovic, JPEG quantization table optimization by guided fireworks algorithm, in: International Workshop on Combinatorial Image Analysis, Springer, 2017, pp. 294–307.

- [36] Pedro Henrique Guimarães Ferreira, Osmar Luiz Ferreira de Carvalho, Eduardo Peixoto, Nature Inspired JPEG Quantization Optimization.
- [37] Stephen P. Brooks, Byron J.T. Morgan, Optimization using simulated annealing, J. R. Stat. Soc.: Ser. D (the Statistician) 44 (2) (1995) 241–257.
- [38] Chen-Hsiu Huang, Ja-Ling Wu, JQF: Optimal JPEG quantization table fusion by simulated annealing on texture images and predicting textures, 2020, arXiv preprint arXiv:2008.05672.
- [39] D. Preethi, D. Loganathan, Quantization table selection using firefly with teaching and learning based optimization algorithm for image compression, in: Handbook of Multimedia Information Security: Techniques and Applications, Springer, 2019, pp. 473–499.
- [40] Rodrigo Morales, Rubén Saborido, Foutse Khomh, Francisco Chicano, Giuliano Antoniol, Earmo: An energy-aware refactoring approach for mobile apps, IEEE Trans. Softw. Eng. 44 (12) (2017) 1176–1206.
- [41] Jetbrains Google, Android Studio.
- [42] Automeris LLC, WebPlotDigitizer.
- [43] James Kennedy, Russell Eberhart, Particle Swarm Optimization (PSO), in: IEEE International Conference on Neural Networks, 1995, pp. 1942–1948.
- [44] Nasir Ahmed, T_ Natarajan, Kamisetty R. Rao, Discrete cosine transform, IEEE Trans. Comput. 100 (1) (1974) 90–93.
- [45] Kamisetty Ramamohan Rao, Jae Jeong Hwang, Techniques and Standards for Image, Video, and Audio Coding, Prentice-Hall, Inc., 1996.
- [46] Xin Yao, Global optimisation by evolutionary algorithms, in: Proceedings of IEEE International Symposium on Parallel Algorithms Architecture Synthesis, IEEE, 1997, pp. 282–291.
- [47] Robert Hooke, Terry A. Jeeves, "Direct search"solution of numerical and statistical problems, J. ACM 8 (2) (1961) 212–229.
- [48] Indraneel Das, John E. Dennis, Normal-boundary intersection: A new method for generating the Pareto surface in nonlinear multicriteria optimization problems, SIAM J. Optim. 8 (3) (1998) 631–657.
- [49] Kalyanmoy Deb, Karthik Sindhya, Tatsuya Okabe, Self-adaptive simulated binary crossover for real-parameter optimization, in: Proceedings of the 9th Annual Conference on Genetic and Evolutionary Computation, 2007, pp. 1187–1194.

- [50] Agoston E. Eiben, James E. Smith, et al., Introduction to Evolutionary Computing, Vol. 53, Springer, 2003.
- [51] Zhi-Hui Zhan, Jun Zhang, Yun Li, Henry Shu-Hung Chung, Adaptive particle swarm optimization, IEEE Trans. Syst. Man Cybern. B 39 (6) (2009) 1362-1381.
- [52] María-Luisa Pérez-Delgado, Color image quantization using the shuffledfrog leaping algorithm, Eng. Appl. Artif. Intell. 79 (2019) 142–158.
- [53] J. Blank, K. Deb, Pymoo: Multi-objective optimization in Python, IEEE Access 8 (2020) 89497-89509.
- [54] Joaquín Derrac, Salvador García, Daniel Molina, Francisco Herrera, A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms, Swarm Evol. Comput. 1 (1) (2011) 3–18.
- [55] David Allen Van Veldhuizen, Multiobjective Evolutionary Algorithms: Classifications, Analyses, and New Innovations, Air Force Institute of Technology, 1999.
- [56] Carlos A. Coello Coello, Margarita Reyes Sierra, A study of the parallelization of a coevolutionary multi-objective evolutionary algorithm, in: Mexican International Conference on Artificial Intelligence, Springer, 2004, pp. 688–697.
- [57] Joshua Knowles, David Corne, Properties of an adaptive archiving algorithm for storing nondominated vectors, IEEE Trans. Evol. Comput. 7 (2) (2003) 100–116.
- [58] Nicola Beume, Carlos M Fonseca, Manuel Lopez-Ibanez, Luis Paquete, Jan Vahrenhold, On the complexity of computing the hypervolume indicator, IEEE Trans. Evol. Comput. 13 (5) (2009) 1075–1082.
- [59] Mauricio Olguin-Carbajal, Enrique Alba, Javier Arellano-Verdejo, Microdifferential evolution with local search for high dimensional problems, in: 2013 IEEE Congress on Evolutionary Computation, 2013, pp. 48–54.
- [60] Viveros-Jimenez Francisco, Mezura-Montes Efren, Gelbukh er, Empirical analysis of a micro-evolutionary algorithm for numerical optimization, Int. J. Phys. Sci. 7 (8) (2012) 1235–1258.
- [61] Ryoji Tanabe, Alex S. Fukunaga, Improving the search performance of SHADE using linear population size reduction, in: IEEE Congress on Evolutionary Computation, IEEE, 2014, pp. 1658–1665.