Lógica Computacional

LEI, 2014/2015
DI-UBI

Aulas Práticas 24

Dedução Natural em Lógica de Primeira Ordem

Provam-se as seguintes afirmações.

1. $\{\forall x\, P(x) \lor \forall x\, Q(x)\} \vdash \forall x\, (P(x) \lor Q(x))$
2. $\vdash (\forall x\, P(x) \land \forall x\, Q(x)) \leftrightarrow \forall x\, (P(x) \land Q(x))$
3. $\{\exists x\, (P(x) \land Q(x))\} \vdash \exists x\, P(x) \land \exists x\, Q(x)$
4. $\vdash (\exists x\, P(x) \lor \exists x\, Q(x)) \leftrightarrow \exists x\, (P(x) \lor Q(x))$
5. $\{\forall x\, (P(x) \to Q(x))\} \vdash \forall x\, P(x) \to \forall x\, Q(x)$
6. $\{\exists y\, \forall x\, \varphi\} \vdash \forall x\, \exists y\, \varphi$
7. $\vdash \exists x\, \neg P(x) \leftrightarrow \neg \forall x\, P(x)$
8. $\vdash \forall x\, \neg P(x) \leftrightarrow \neg \exists x\, P(x)$
9. $\vdash \exists x\, \varphi \leftrightarrow \neg \forall x\, \neg \varphi$
10. $\vdash \forall x\, \varphi \leftrightarrow \neg \exists x\, \neg \varphi$
11. $\vdash \forall x\, \varphi \land \psi \leftrightarrow \forall x\, (\varphi \land \psi)$, se $x \notin \mathsf{VL}(\psi)$
12. $\vdash \forall x\, \varphi \lor \psi \leftrightarrow \forall x\, (\varphi \lor \psi)$, se $x \notin \mathsf{VL}(\psi)$
13. $\vdash \exists x\, \varphi \land \psi \leftrightarrow \exists x\, (\varphi \land \psi)$, se $x \notin \mathsf{VL}(\psi)$
14. $\vdash \exists x\, \varphi \lor \psi \leftrightarrow \exists x\, (\varphi \lor \psi)$, se $x \notin \mathsf{VL}(\psi)$
15. $\vdash \forall x\, (\psi \to \varphi) \leftrightarrow \psi \to \forall x\, \varphi$, se $x \notin \mathsf{VL}(\psi)$
16. $\vdash \exists x\, (\psi \to \varphi) \leftrightarrow \psi \to \exists x\, \varphi$, se $x \notin \mathsf{VL}(\psi)$
17. $\vdash \forall x\, (\varphi \to \psi) \leftrightarrow \exists x\, \varphi \to \psi$, se $x \notin \mathsf{VL}(\psi)$
18. $\vdash \exists x\, (\varphi \to \psi) \leftrightarrow \forall x\, \varphi \to \psi$, se $x \notin \mathsf{VL}(\psi)$