On the Use of Evolutionary Algorithms for Reactive Power Compensation in Electrical Distribution Networks – Experiments on a Case Study

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1 Introduction

In the electrical distribution network, reactive power compensation aims at guarantying an efficient delivery of active power to loads, releasing system capacity, reducing system losses, and improving system power factor and bus voltage profile. The achievement of these aims depends on the sizing and location of shunt capacitors (sources of reactive power). Operational, economical and quality of service objectives must be weighed by decision-makers / planning engineers to select acceptable solutions having in mind their practical implementation. Multi-objective models enable to grasp the conflicting nature of the objectives and the tradeoffs to be made in order to identify satisfactory compromise solutions by providing a basis to rationalize the comparison between non-dominated solutions.

The problem of optimal capacitor placement in radial electrical distribution systems is formulated considering two objective functions: minimizing capacitor installation cost and minimizing system losses. Constraints are related with requirements of acceptable node voltage profile (quality of service), power flow (physical laws in electrical networks), and impossibility of capacitor location at certain nodes (technical restrictions). The aim is to identify non-dominated solutions to the multi-objective model, involving the determination of the size and locations of capacitor banks to be installed.

A large range of models and methodological approaches have been proposed in the scientific literature devoted to the reactive power compensation problem. Approaches to tackle this problem included analytical methods, mathematical programming algorithms, and heuristics and meta-heuristics [4].

An evolutionary approach has been developed to compute non-dominated solutions to the problem of reactive power compensation in electrical radial distribution networks. The ability to work in each generation with a population of potential solutions makes evolutionary approaches well suited for multi-objective optimization problems in which a set of non-dominated solutions must be identified rather than a single optimal solution [1, 2]. Moreover, this combinatorial problem is very complex to be tackled by mathematical programming tools since, besides its multi-objective nature, it is non-linear with continuous, integer and binary variables. An elitist strategy has been implemented aimed at increasing the performance, both accelerating the convergence speed towards the non-dominated frontier and ensuring the solutions attained are indeed non-dominated ones and are well-spread over the frontier. This is an important issue in real-world combinatorial problems since it is necessary to provide the decision maker (DM) with well-distributed and diverse solutions for a well-informed final decision to be made upon. Illustrative results are presented which have been obtained with the evolutionary approach applied to the model instantiated with real-world data of a Portuguese distribution network.

2 A multi-objective approach for reactive power compensation

The mathematical model is similar to the one described in detail in [5]. Two conflicting objective functions: minimizing (resistive) losses and minimizing the installation costs of new sources of reactive power. Quality of service requirements associated with an acceptable voltage profile in load buses are included as constraints.

The real-valued decision variables refer to load (real power and reactive power) in buses, voltage magnitude in nodes, real power and reactive power flowing feeders, reactive power injections from capacitors located in nodes. Binary decisions variable refer to the decisions of installing given capacitor at a certain candidate node.

A set of recursive equations describe the physical requirements associated with power flow through each branch in a radial electrical distribution system (Figure 1; SE is the substation). The procedure used for this purpose for load flow computation is adapted to radial networks, such as the ones used in the Portuguese electrical distribution system [5]. Others sets of constraints impose that, at most, one capacitor can be placed in each node. Upper and lower bounds of node voltage magnitude are imposed, which are related with quality of service.

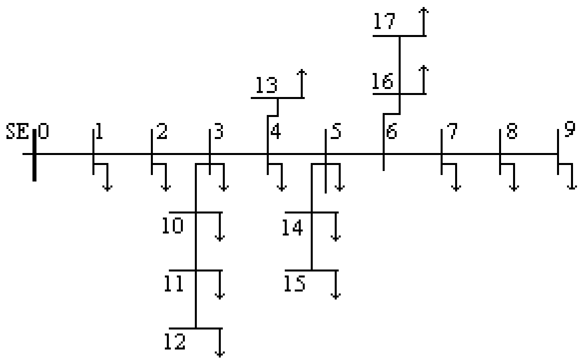

Figure 1: Example of a radial electrical distribution system.

In the implementation of the genetic algorithm devoted to the reactive power compensation problem, an elitist strategy is used with a secondary population of constant size, consisting of the following main steps:

- the fitness of the individuals composing the main population is computed;
- from the main population (consisting of POP individuals) POP-E individuals are selected by using a tournament technique;
- a new population is formed by the POP-E offspring generated by crossover and mutation;
- E individuals (elite) are randomly selected from the secondary population;
- the evaluation of individuals by a dominance test is carried out, which defines an approximation to the non-dominated frontier;
- the non-dominated solutions are computed and they are processed to update the secondary population using a sharing technique (aimed at favouring a well-spread distribution of the secondary population throughout the objective space), if necessary.

The population used in the algorithm implementation consists of individuals represented by an array of NN integer values (NN being the number of network nodes where it is possible to install a new capacitor or change the capacity of a capacitor already installed). The index of the array corresponds to a network node and the value therein denotes the type of capacitor to install in that node (0 denotes no capacitor to be installed).

The computation of the fitness of each solution involves determining various solution fronts in the following way (see [3], for a similar approach):

- A niche is defined by a radius *dist* around a solution, where *dist* is the maximum distance between solutions necessary to obtain a well-spread front and is equal to $\sqrt{2}/$ POP; POP is the size of the main population and $\sqrt{2}$ is the normalized distance between the pseudo-solutions obtained by considering the best and the worst values for each objective function in the main population;
- The first front consists of all non-dominated solutions, a maximum fitness value equal to POP^2 being assigned to them;
- This fitness value of each one of these solutions is subtracted by the number of solutions belonging to a niche defined by a radius *dist* around that solution;

- The solutions in the first front are temporarily ignored and the remaining feasible solutions (the dominated solutions) are processed by applying them a dominance test (the non-dominated solutions will belong to the second front);
- The maximum fitness value of the current front is obtained by subtracting POP to the maximum fitness value of the previous front, which is assigned to the solutions of the current front;
- For each solution in the current front, the fitness value is subtracted by the number of solutions belonging to the same niche;
- This process continues until all feasible solutions are assigned a fitness value;
- The same process is repeated for the non-feasible solutions until all non-feasible solutions are assigned a fitness value.

The sharing mechanism used in updating the secondary population uses a niche scheme whose radius is a dynamic value. This mechanism is applied after computing all non-dominated solutions candidates for the secondary population. These are all the solutions already belonging to the secondary population which are not dominated by any solution in the main population and the non-dominated solutions of the main population which are not dominated by solutions in the secondary population. This mechanism is only applied in case the number of solutions candidates for the secondary population (NCPS) is greater than the size of this population (NPS). The sharing mechanism consists in the following steps:

1. Insert the two extreme solutions (those with the best values for each objective function);
2. Compute the first niche radius (*dist*) as the ratio: normalized distance between extreme solutions / NPS (that is, $\sqrt{2}$/ NPS);
3. Insert solutions located at a distance greater than *dist* from the ones already belonging to the secondary population;
4. Update the value of niche radius, *dist*, by reducing it by 10%;
5. If the secondary population is not complete then return to step 3.

The strategy used to determine the initial population consists of randomly generating non-dominated feasible solutions only. This strategy produced better results when compared with strategies that randomly generate solutions of any type (feasible or non-feasible) or feasible solutions (dominated or non-dominated) only.

Crossover is always present (pc = 1). Two-point crossover has been used, because it produced better results than one-point and uniform crossover. The two crossover points are selected randomly with the restriction that they have to be apart for at least 1/4 of chromosome size.

The capacitor type is indexed by an index ranging from 0 to J (where 0 means no capacitor; that is, J different capacitor size can be installed). The mutation consists in modifying (with a probability *pm*) the current index value to one of other possible values.

The algorithm associated with this approach consists in the following steps:

1. Initialization: randomly generate the initial population with POP non-dominated solutions;

2. Evaluation: compute the fitness value of each individual in the initial population;
3. Determine the initial secondary population of maximum size NPS from the initial population: if NPS ≥ POP then copy all non-dominated solutions from the initial population to the secondary population; else apply the sharing mechanism to the initial population to select NPS solutions;
4. Current population ← initial population;

Repeat

5. Build up the (main) population associated with the next generation of size POP:
   a) Introduce directly E individuals from the secondary population (elite) into the main population;
   b) Select 2 individuals of the current population by tournament (in each tournament, 10% of the individuals in the current population are used to produce the selected one);
   c) Apply genetic operators crossover and mutation to the 2 individuals selected;
   d) Insert the new individuals into the main population;
   e) If the main population does not yet contain POP individuals then return to step b);
6. Evaluation: apply the dominance test and compute the fitness value of each individual in the main population;
7. Determine the NCPS solutions candidates to becoming part of the secondary population;
8. Update the secondary population: if NPS ≥ NCPS then copy all candidate non-dominated solutions to the secondary population; else apply the sharing mechanism to all solutions found in step 7 to select NPS solutions;
9. Current population ← main population;

Until the pre-specified number of iterations is attained.

3 Results of the experiments on a real-world case study

This methodology to characterize the Pareto Optimal front has been applied to an actual Portuguese radial distribution system in a sparsely populated area (which imposes heavier operating conditions to the network). Several runs have been done with different sets of parameters. The best results were obtained with: POP = 30, NPS = 40, E = 4, 7500 generations, pm = 0.1, and pc = 1.

Figure 2(a) displays the Pareto front, in the objective function space, regarding the final secondary population (that is, the output of the process to be presented to DMs/planning engineers). Figure 2(b) enables a comparison to be made between the final secondary population and the initial population. Each solution is associated with a compensation scheme defined by: number and size of the capacitors, the network nodes where they are installed, and the corresponding cost and resistive losses.

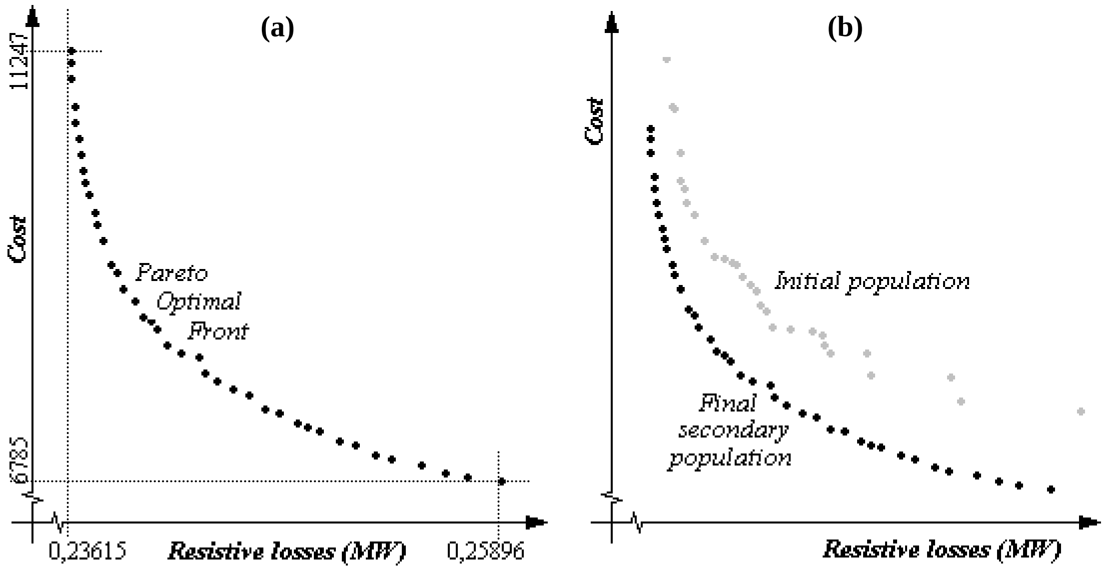

Figure 2: Final secondary population with 40 solutions (a) and *versus* initial population (b).

In order to fine tune the algorithm to the characteristics of the case study, some variants of the techniques used were also implemented and tested. In particular, experiments have been made using different strategies to build the initial population and distinct types of crossover.

For this specific problem, three crossover types were analyzed: (A) one-point, (B) two-point, and (C) uniform with a randomly generated mask. The experiments carried out indicate that the two-point crossover produces better results than the other approaches (Figure 3).

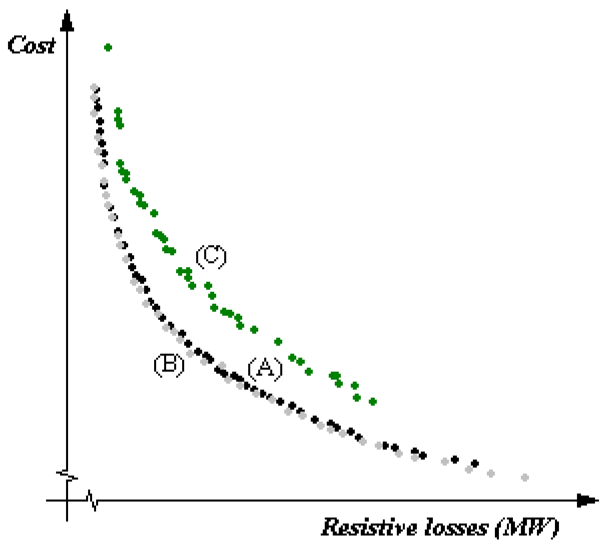

Figure 3: Final secondary populations associated with the best results for each crossover type.

Three strategies to determine the initial population were analyzed which are related to the type of solutions that belong to the population: (A) feasible and non-feasible solutions; (B) feasible (dominated and non-dominated) solutions only; (C) feasible non-dominated solutions only.

The initial populations consisting of feasible non-dominated solutions only (type C) produced the best results. The Pareto Optimal front determined by the algorithm using this type of initial population dominates the fronts determined by using initial populations of types (A) and (B), as shown in Figure 4.

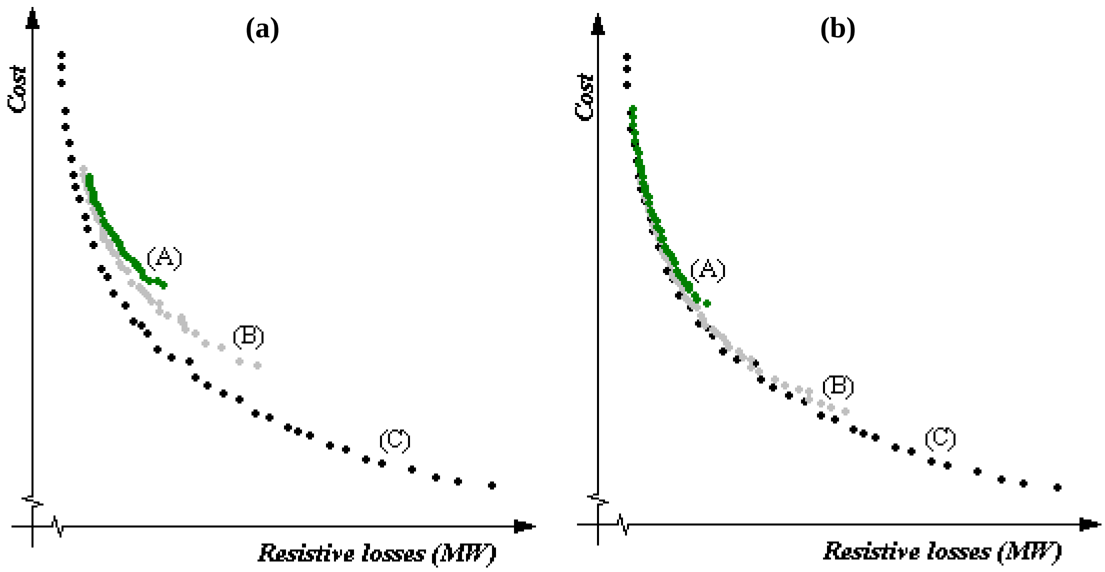

Figure 4: Final secondary populations for the same set of parameters (a) and the best results (b).

Figure 4(a) shows the three Pareto Optimal fronts associated with the three types of initial population (for the same set of parameters: POP = 30, NPS = 40, E = 4, number of generations = 7500, pm = 0.1, and pc = 1).

Figure 4(b) displays the three Pareto Optimal fronts associated with the best results for each type of initial population (for all sets of parameters tested):

- type A: POP = 100, NPS = 40, E = 6, number of generations = 10000, pm = 0.1, and pc = 1;
- type B: POP = 40, NPS = 50, E = 4, number of generations = 7500, pm = 0.1, and pc = 1;
- type C: POP = 30, NPS = 40, E = 4, number of generations = 7500, pm = 0.1, and pc = 1.

For the 94 node distribution network, the run times (Pentium III, 1 GHz, 512 RAM) associated with each type of initial population are the following (two-point crossover was adopted for all runs, and 12 run averages are presented):

- using the same set of parameters: 51 sec. (A), 57 sec. (B), and 387 sec. (C) (Figure 4(a));
- for the best results obtained: 267 sec. (A), 100 sec. (B), and 387 sec. (C) (Figure 4(b)).

However, though the run times associated with initial population of type (C) have been the longest, the average run time for this type was 260 sec. (the best run time was 96 sec. and the worst run time was 406 sec.). The run times include the load flow calculation, which are adapted to radial networks in electrical distribution systems.

5 Conclusions

An evolutionary approach consisting of an elitist genetic algorithm with secondary population has been presented for providing decision support in the location and sizing of capacitors for reactive power compensation in electrical radial distribution networks. The algorithm is aimed at characterizing the Pareto Optimal frontier by computing well-distributed solutions which are representative of distinct compromises to be faced by DMs/planning engineers in the selection of practical plans.

The results of a large set of experiments have been briefly reported regarding the performance of the evolutionary approach, namely concerning its behaviour with respect to different ways of building the initial population and distinct crossover types. These results enabled to fine tune the methodological approach for achieving a better performance in real-world studies in the Portuguese distribution network.

Acknowledgments

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References

[1] Coello, C., D. Veldhuizen, G. Lamont (2002): *Evolutionary Algorithms for Solving Multi-Objective Problems*. Kluwer Academic Publishers.

[2] Deb, K. (2001): *Multi-Objective Optimization Using Evolutionary Algorithms*. Wiley.

[3] Deb, K., A. Pratap, S. Agarwal, T. Meyarivan (2002): “A fast and elitist multi-objective genetic algorithm: NSGA-II”. *IEEE Trans. Evolutionary Computation*, 6 (2), 182-197.

[4] Delfanti, M., G. Granelli, P. Marannino, M. Montagna (2000): “Optimal capacitor placement using deterministic and genetic algorithms”. *IEEE Trans. Power Systems*, 15 (3), 1041-1046.

[5] Pires, D., A. Martins, C. H. Antunes (2004): “A multiobjective model for VAR planning in radial distribution networks based on Tabu Search”. IEEE *Trans. Power Systems*(in print).