Computação Gráfica

5385: Licenciatura em Engenharia Informática

Chap. 8 — Rasterization

Rasterization

Outline

...:

- Raster display technology.
- Basic concepts: pixel, resolution, aspect ratio, dynamic range, image domain, object domain.
- Rasterization and direct illumination.
- Graphics primitives and OpenGL.
- Geometry representations: explicit, parametric and implicit forms.
- Rasterization algorithms for straight line segments, circles and ellipses.
- Rasterization algorithms for triangles and polygons.
- Rasterization versus shading.

Raster display

Definition:

- Discrete grid of elements (frame buffer of pixels).
  - Shapes drawn by setting the “right” elements
  - Frame buffer is scanned, one line at a time, to refresh the image (as opposed to vector display)

Properties:

- Difficult to draw smooth lines
- Displays only a discrete approximation of any shape
- Refresh of entire frame buffer

Terminology

Pixel: Picture Element

- Smallest accessible element in picture.
- Usually rectangular or circular.

Aspect Ratio:

- Ratio between physical dimensions of pixel (not necessarily 1).

Dynamic Range:

- Ratio between minimal (not zero!) and maximal light intensity emitted by displayed pixel

Resolution:

- Number of distinguishable rows and columns on a device measured in:
  - Absolute values (nxm)
  - Relative values (e.g., 300 dpi)
- Usually rectangular or circular.

Screen space:

- Discrete 2D Cartesian coordinate system of screen pixels.

Object space:

- Continuous 3D Cartesian coordinate system of the domain or scene or the objects live in.

SCAN CONVERSION

Scan conversion / rasterization (for *direct illumination*)

Definition:

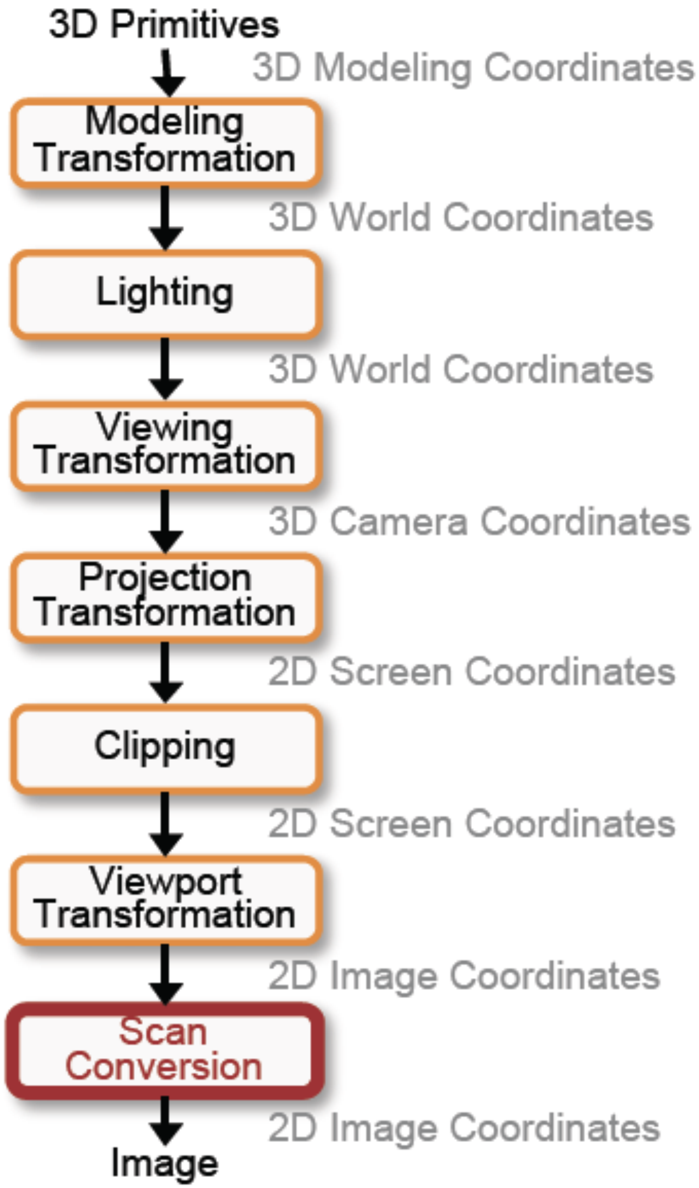
- The process of converting geometry into pixels.
- Final step in pipeline: *rasterization* (*scan conversion*)
- From screen coordinates (float) to pixels (int)
- Writing pixels into frame buffer.

Scan conversion:

- Figuring out which pixels to turn on.

Shading:

- Determine a color for each filled pixel.



Graphics primitives

OpenGL Primitive Taxonomy:

- Point: `POINTS`
- Line: `LINES`, `LINE_STRIP`, `LINE_LOOP`
- Triangle: `TRIANGLES`, `TRIANGLE_STRIP`, `TRIANGLE_FAN`
- Polygon: `QUADS`, `QUAD_STRIP`, `POLYGON`

Other Primitives:

- Arc
- Circle
- Ellipsis
- Generic Curves

How is each geometric primitive really drawn on screen?

Geometric representations for lines in IR^2

Explicit form:

$$y = f(x) = mx + b$$

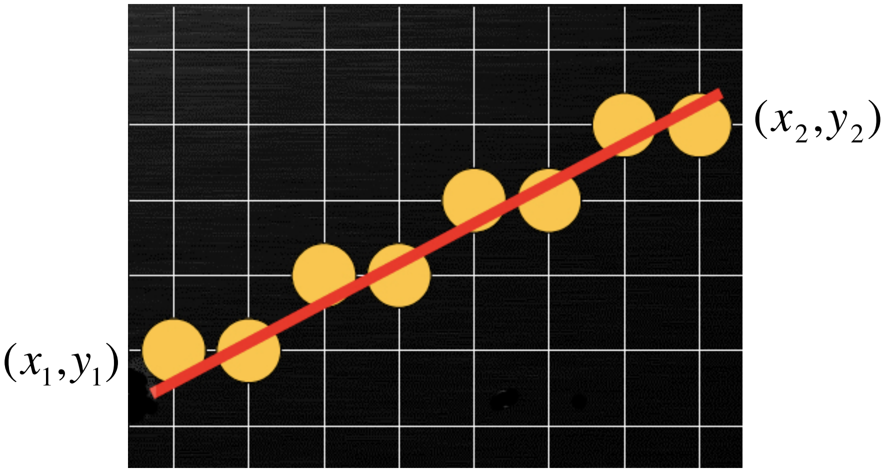
Implicit form:

$$f(x,y) = Ax + By + C = 0$$

Parametric form:

$$x = x(t) = m_0 t + b_0$$
$$y = y(t) = m_1 t + b_1$$

Scan converting lines

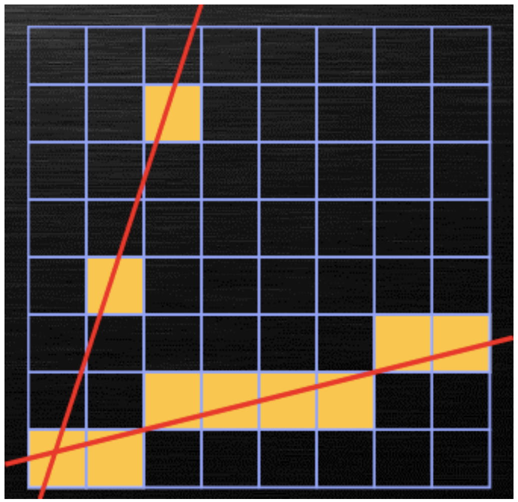


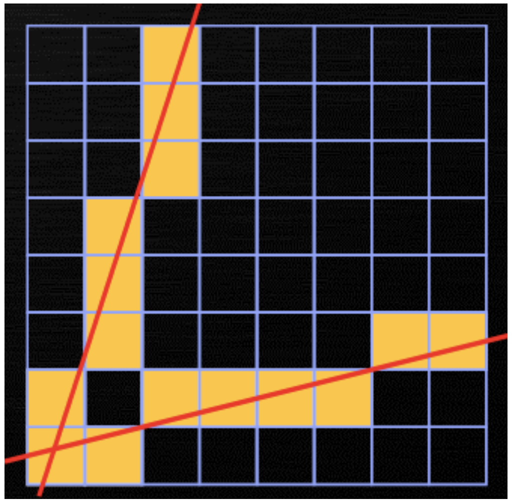
Example:

– Draw from (x_1,y_1) to (x_2,y_2)

Correctness/quality issues:

– Gaps exist for line with slope m>1





Direct scan conversion

Explicit form:

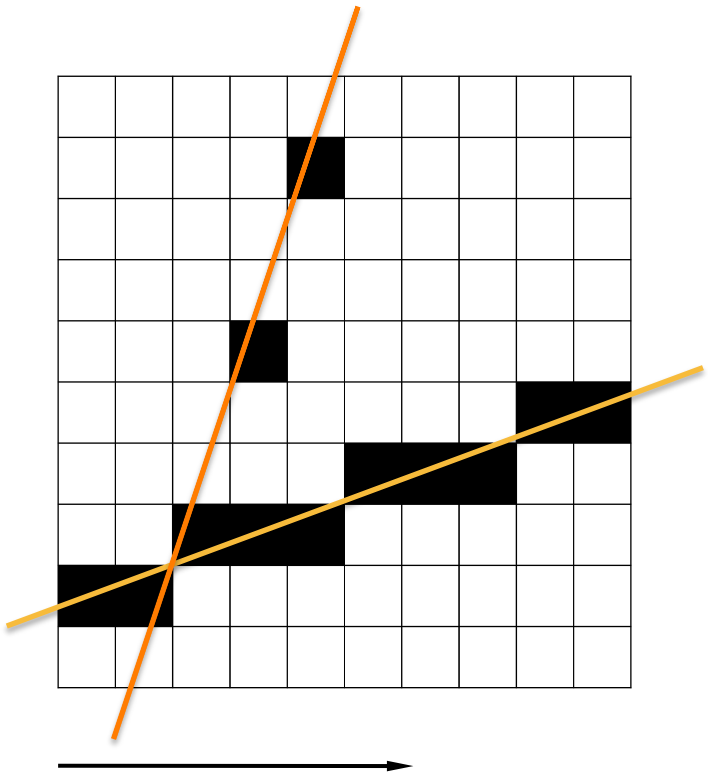
- $y=mx+b$, where $m=(y_{i+1}-y_i)/(x_{i+1}-x_i)=\Delta y/\Delta x$ and $0 \le m \le 1$ (1^{st}, 4^{th}, 5^{th} and 8^{th} octants)
- What else?

Key idea:

- Increment x from x_i to x_f and calculate the corresponding value $y=mx+b$

Drawbacks:

- Gaps when $m>1$. The solution is to increment y instead of x when $m>1$.
- Floating-point computations: floating-point multiplication and addition for every step in *x*.

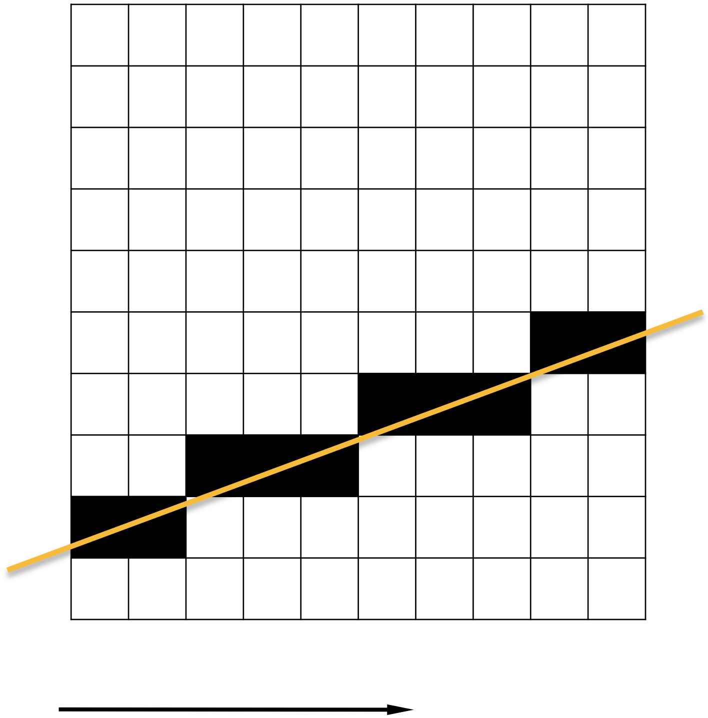


increments in x
(fails when m>1)

Direct scan conversion (cont'd)

Algorithm (m<1):

- $m=(y_f-y_i)/(x_f-x_i)$;
- $b=y_i-m^*x_i$;
- x=xi; y=yi;
- DrawPixel(x,y);
- for ($x=x_i+1$; $x<=x_f$; x++).
  - y=m*x+b;
  - DrawPixel(x,y);



increments in x
(m<1)

DDA algorithm (Digital Differential Analyser)

Explicit form:

– $y=mx+b$, where $m=(y_{i+1}-y_i)/(x_{i+1}-x_i)=\Delta y/\Delta x$ and $0\leq m\leq 1$ (1st, 4th, 5th and 8th octants)

Key idea:

– Increment x from x_i to x_f and calculate the corresponding value y:

– Current pixel: $y_i=mx_i+b$

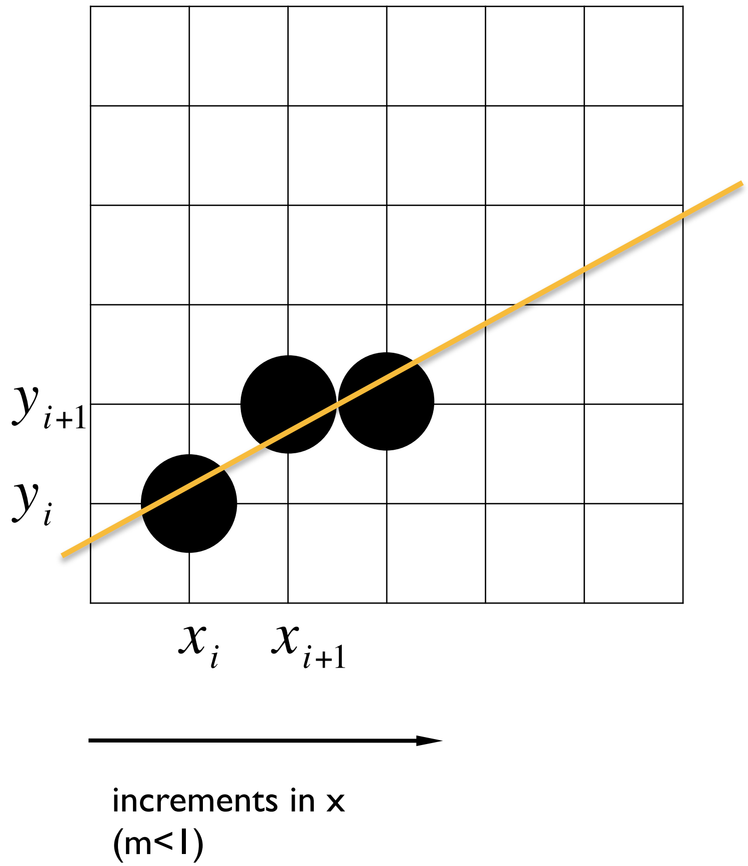
– Next pixel:

- $y_{i+1}=mx_{i+1}+b=m(x_i+1)+b=y_i+m$
- Draw pixel (x_{i+1},y_{i+1}), where y_{i+1}=ROUND(y_{i+1})

Drawbacks:

– Gaps when m>1. In this case, increment y.

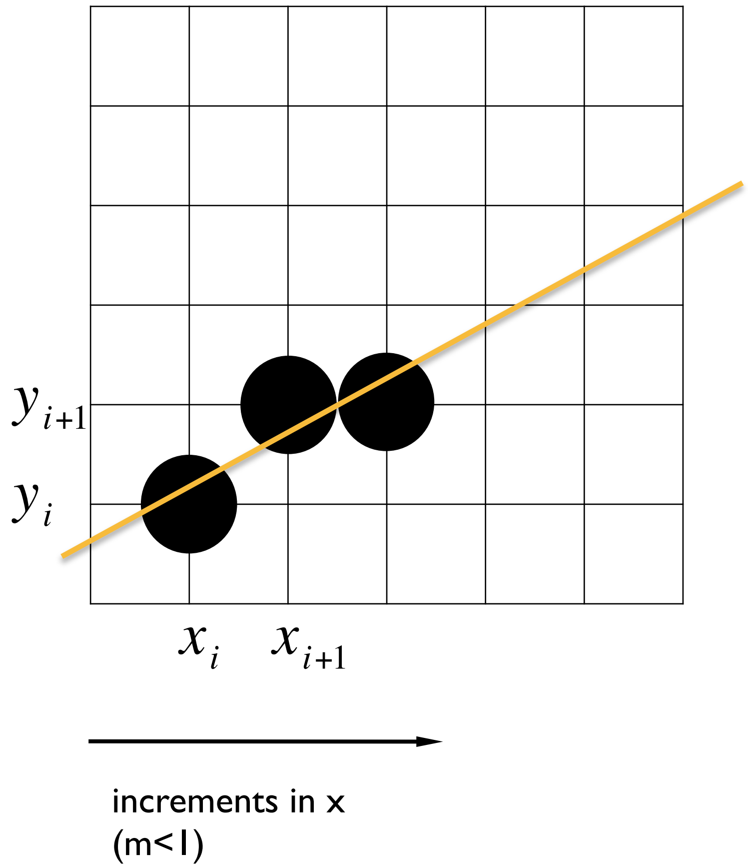
– Floating-point arithmetic: a floating-point addition and a round operation.



DDA algorithm (Digital Differential Analyser)

Algorithm (m<1):

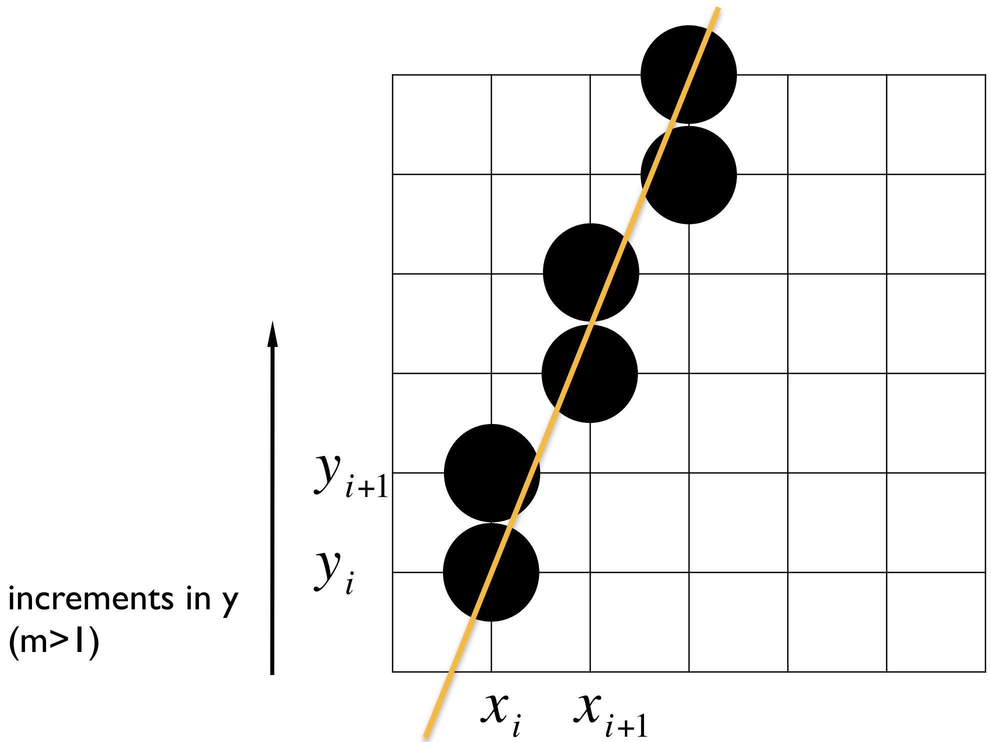
- $m=(y_f-y_i)/(x_f-x_i)$;
- x=xi; y=yi;
- DrawPixel(x,y);
- for ($x=x_i+1$; $x<=x_f$; x++).
  - y=y+m;
  - DrawPixel(x,y);



DDA algorithm (cont'd)

Algorithm (m>1):

- $m=(y_f-y_i)/(x_f-x_i)$;
- x=xi; y=yi;
- DrawPixel(x,y);
- for $(y=y_i+1;\ y<=x_y;\ y++)$.
  - x=?; (find the expression!)
  - DrawPixel(x,y);



Bresenham algorithm

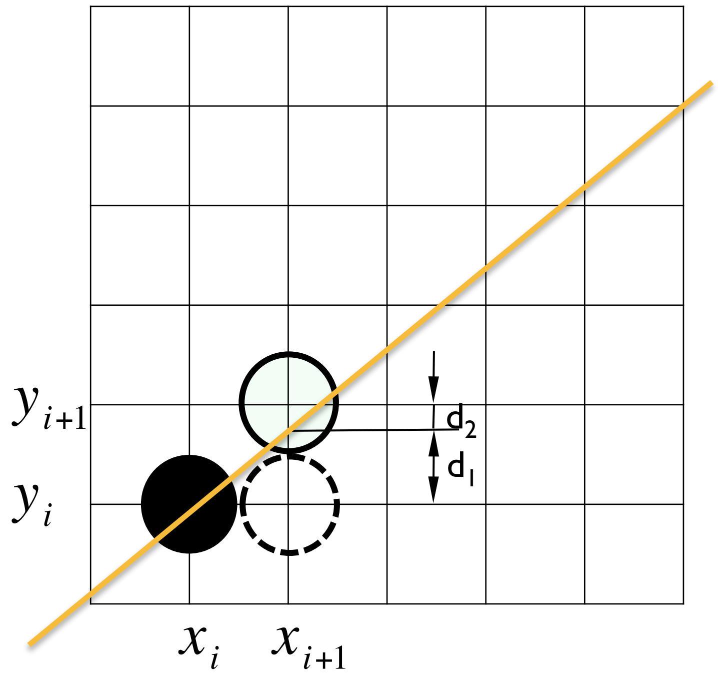
Bresenham, J.E. *Algorithm for computer control of a digital plotter, IBM Systems Journal, January 1965, pp. 25-30.*

Explicit form:

- $y=mx+b$, where $m=\Delta y/\Delta x$ and $0 \leq m \leq 1$

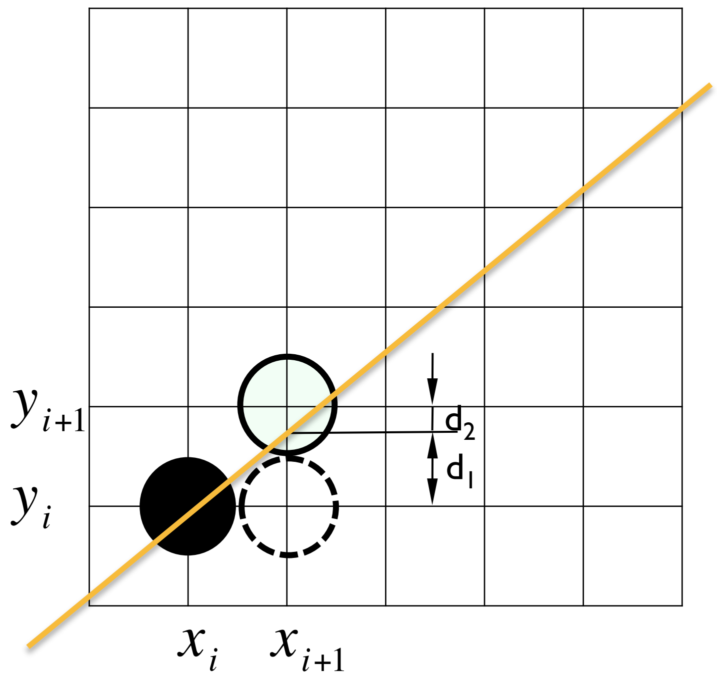
Key idea:

- Increment x from x_i to x_f and calculate the corresponding value y.
- Current pixel: (x_i,y_i)
- Next pixel: either (x_{i+1},y_i) or (x_{i+1},y_{i+1})
  - $d_1=y-y_i=mx_{i+1}+b-y_i=m(x_i+1)+b-y_i$
  - $d_2=y_{i+1}-y=y_i+1-y=y_i+1-m(x_i+1)+b$
  - $\Delta d=d_1-d_2=2m(x_i+1)-2y_i+2b-1$
  - If $\Delta d>0$ choose higher pixel (x_{i+1},y_{i+1})
  - If $\Delta d \leq 0$ choose lower pixel (x_{i+1},y_i)



Exact y-coordinate value $y=y_i+d_1$ of straight line at $x=x_{i+1}$

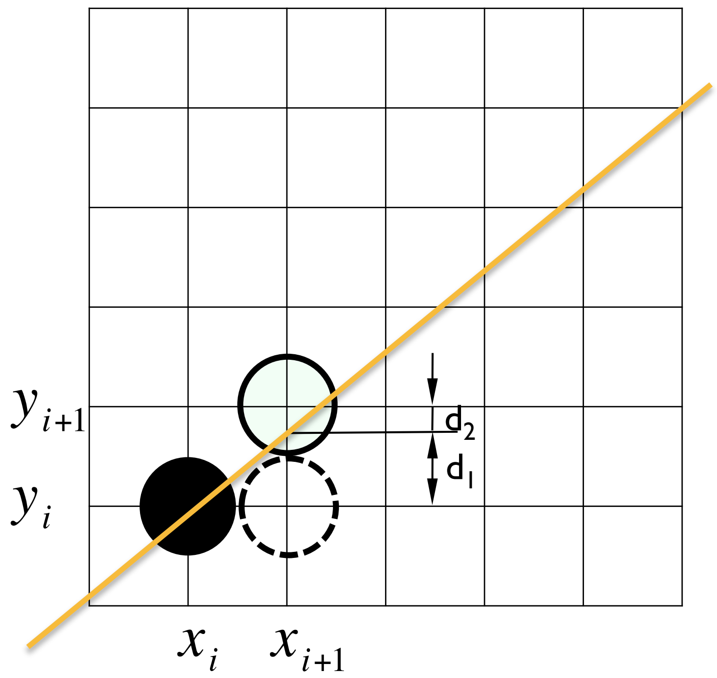
Bresenham algorithm (cont'd)



Integer arithmetic (?):

- From triangle similarity, we know that
  - $m=\Delta y/\Delta x=d_1/(x_{i+1}-x_i)=d_1$
  - $d_2=1-d_1=1-m$
- Hence
  - $d_1-d_2=2m-1$
- To take advantage of integer arithmetic, we use the following decision parameter at the first pixel (x_i,y_i) to choose which is the next pixel:
  - $\boxed{p_i=\Delta x(d_1-d_2)=2\Delta y-\Delta x}$
- But, in general terms, and using d_1 and d_2 in the previous page, we have:
  - $p_i=\Delta x(d_1-d_2)=2\Delta y.x_i-2\Delta x.y_i+K$, where K is a constant
- Consequently, the decision parameter at (x_{i+1},y_{i+1}) will be:
  - $p_{i+1}=2\Delta y.x_{i+1}-2\Delta x.y_{i+1}+K$ or
  - $\boxed{p_{i+1}=p_i+2\Delta y(x_{i+1}-x_i)-2\Delta x(y_{i+1}-y_i)}$ (note that $x_{i+1}-x_i=1$)

Bresenham algorithm (cont'd)



Algorithm:

```
void Bresenham (int xi, int yi, int xf, int yf)
{
        int x,y,dx,dy,p;
        x = xi; y = yi;
        p = 2 * dy - dx;
        for(x=xi; x<=xf; x++)
        {
                DrawPixel (x,y);
                if (p> 0)
                {
                        y = y + 1;
                        p = p - 2 * dx;
                }
                p= p + 2 * dy;
        }
}
```

Midpoint algorithm

Bresenham, J.E. *Algorithm for computer control of a digital plotter, IBM Systems Journal, January 1965, pp. 25-30.*

Implicit form:

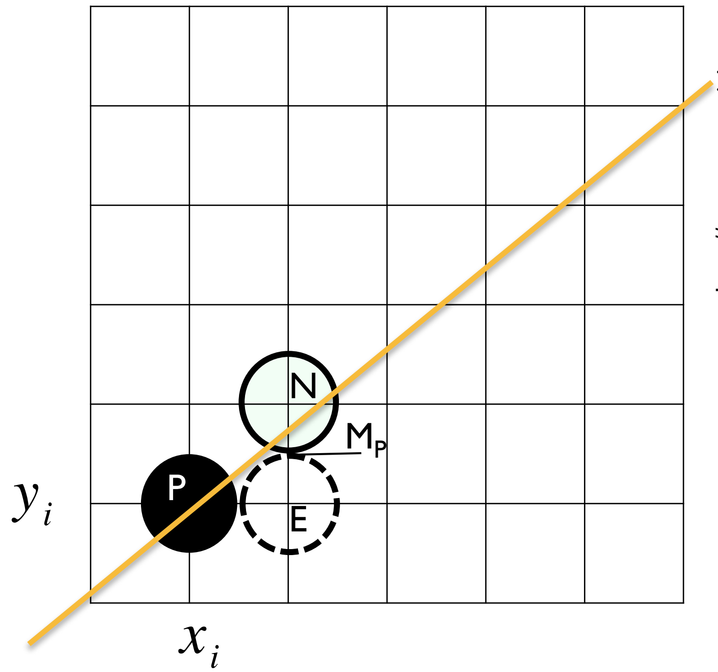
- $f(x,y)=Ax+By+C=0$

Key idea:

- Starting from $y=mx+b$, where $m=\Delta y/\Delta x$ and $0 \leq m \leq 1$, we have:

  $$f(x,y)=\Delta y.x-\Delta x.y+b.\Delta x=0$$

  with $A=\Delta y$, $B=-\Delta x$, and $C=b.\Delta x$
- Current pixel: (x_i,y_i)
- Next pixel: either (x_{i+1},y_i) or (x_{i+1},y_{i+1})
  - Let the decision parameter $p_i=f(M_P)=f(x_i+1,y_i+1/2)$
  - If $p_i<0$ choose higher pixel (x_{i+1},y_{i+1}) at N
  - If $p_i \geq 0$ choose lower pixel (x_{i+1},y_i) at E



Current pixel $P(x_i,y_i)$

Midpoint algorithm (cont'd)

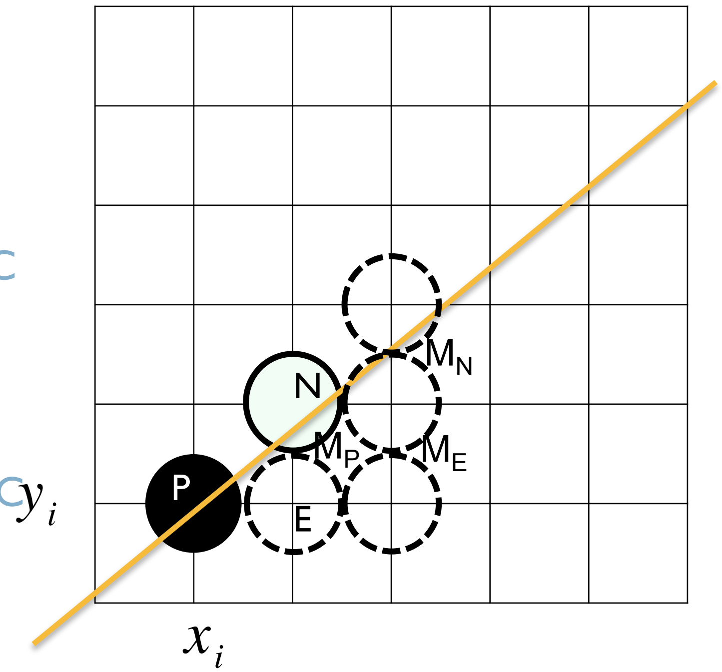
Let us now determine the relation between the function values at consecutive midpoints:

Key idea (cont'd):

- $p_i=f(M_P)=f(x_i+1,y_i+1/2)=A.(x_i+1)+B.(y_i+1/2)+C$
- If E is chosen:
  - $p_{i+1}=f(M_E)=f(x_i+2,y_i+1/2)=A.(x_i+2)+B.(y_i+1/2)+C$

    $=p_i+A=p_i+\Delta y$
- If N is chosen:
  - $p_{i+1}=f(M_N)=f(x_i+2,y_i+3/2)=A.(x_i+2)+B.(y_i+3/2)+C$

    $=p_i+A+B=p_i+\Delta y-\Delta x$



Current pixel:
E (East) or
N (Nord-East)

Integer arithmetic (?):

- Initial decision parameter:
  - $p_i=f(M_P)=f(x_i+1,y_i+1/2)=A.(x_i+1)+B.(y_i+1/2)+C$

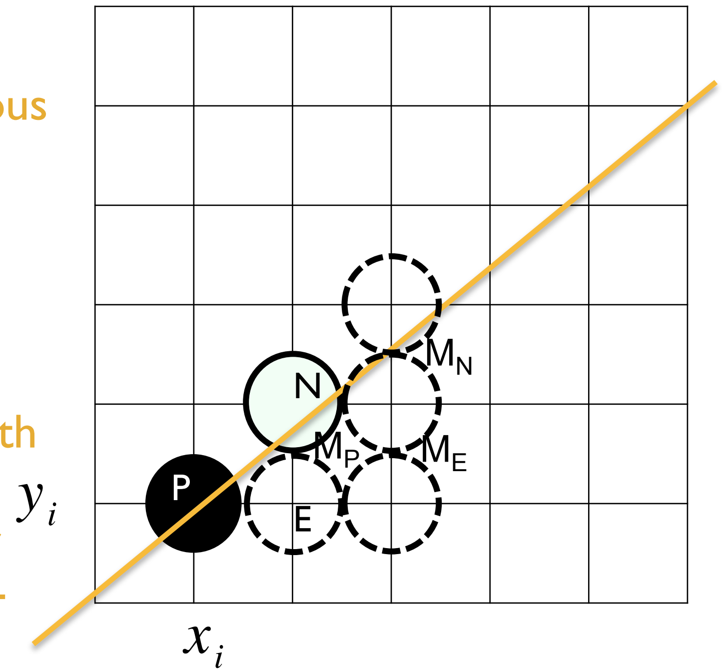
    $=f(P)+A+B/2=f(P)+\Delta y-\Delta x/2=\Delta y-\Delta x/2$

Multiplying the decision parameter by 2 we realize that we obtain exactly the Bresenham algorithm given before.

General Bresenham's algorithm for lines

To generalize lines with arbitrary slopes:

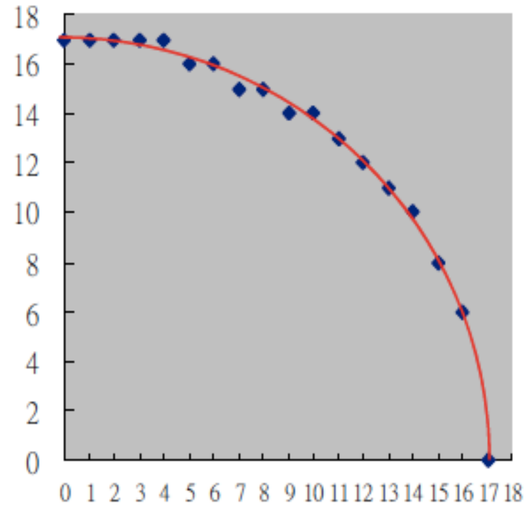
- We need to consider symmetry between various octants and quadrants.
- For m>1, interchange roles of x and y, that is step in y direction, and decide whether the x value is above or below the line.
- If m>1, and right endpoint is the first point, both x and y decrease. To ensure uniqueness, independent of direction, always choose upper (or lower) point if the line go through the mid-point.
- Handle special cases without invoking the algorithm: horizontal, vertical and diagonal lines



Scan converting circles

Explicit form: $y = f(x) = \pm\sqrt{R^2 - x^2}$

– Usually, we draw a quarter circle by incrementing x from 0 to R in unit steps and solving for +y for each step.



gap problem

Parametric form: $\begin{cases} x = R\cos\theta \\ y = R\sin\theta \end{cases}$

– Done by stepping the angle from 0 to 90°.

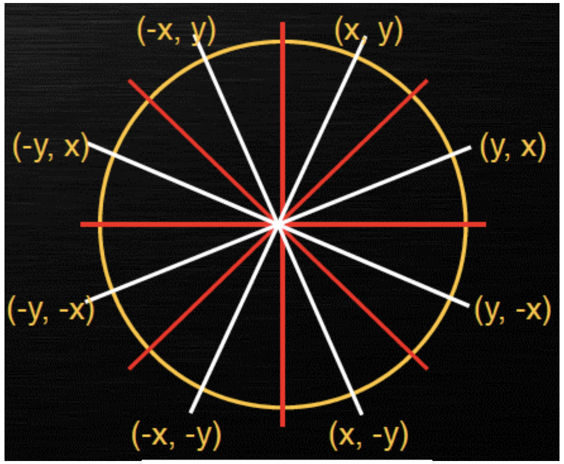
– Solves the gap problem of explicit form.

Implicit form: $f(x,y) = x^2 + y^2 - R^2 = 0$

– If f(x,y)=0, then it is on the circle;

– If f(x,y)>0, then it is outside the circle;

– If f(x,y)<0, then it is inside the circle.



8-way symmetry

Midpoint circle algorithm

J.E. Bresenham. A linear algorithm for incremental digital display of circular arcs. *Communications of the ACM, 20(2): 100-106, 1977.*

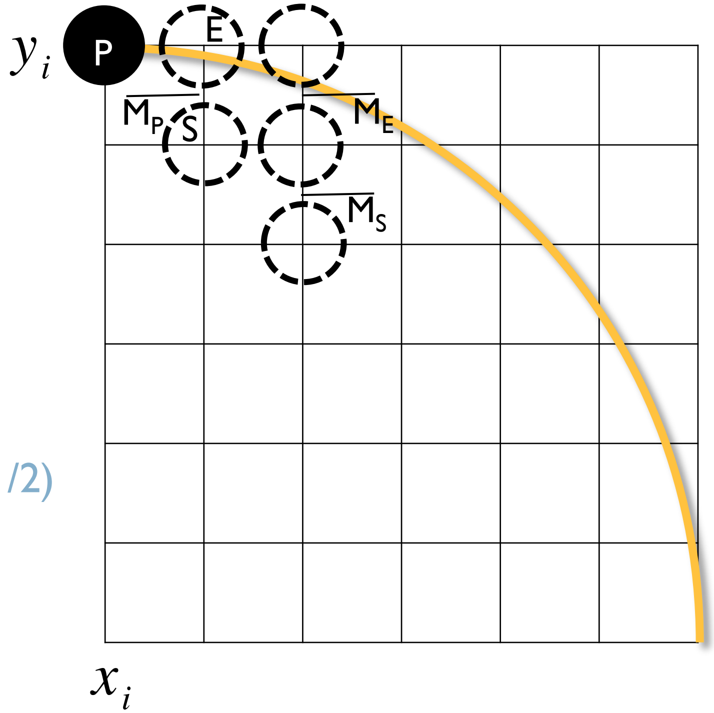
http://www.mat.univie.ac.at/~kriegl/Skripten/CG/node25.html

Implicit form:

- $f(x,y)=x^2+y^2-R^2=0$

Key idea:

- Current pixel: $P(x_i,y_i)$
- Next pixel: either (x_{i+1},y_i) or (x_{i+1},y_{i-1})
  - Let the decision parameter $p_i=f(M_P)=f(x_i+1,y_i-1/2)$
  - If $p_i<0$ choose higher pixel (x_{i+1},y_i) at E
  - If $p_i\geq 0$ choose lower pixel (x_{i+1},y_{i-1}) at S



Current pixel $P(x_i,y_i)$

Midpoint circle algorithm (cont'd)

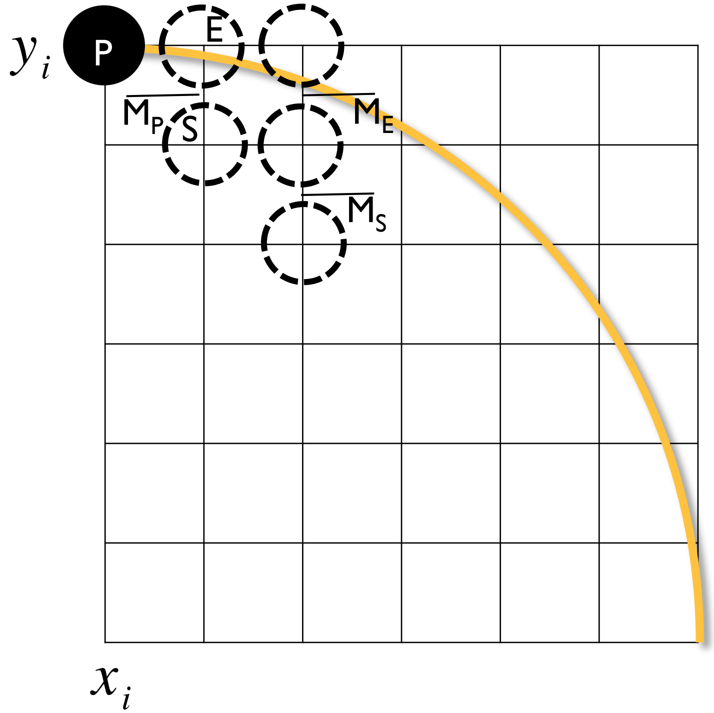
Let us now determine the relation between the function values at consecutive midpoints:

Key idea (cont'd):

- $p_i = f(M_P) = f(x_i+1, y_i-1/2) = (x_i+1)^2 + (y_i-1/2)^2 - R^2$
- If E is chosen:
  - $p_{i+1} = f(M_E) = f(x_i+2, y_i-1/2) = (x_i+2)^2 + (y_i-1/2)^2 - R^2$
    $= p_i + (2x_i+3)$
- If S is chosen:
  - $p_{i+1} = f(M_S) = f(x_i+2, y_i-3/2) = (x_i+2)^2 + (y_i-3/2)^2 - R^2$
    $= p_i + (2x_i-2y_i+5)$

Integer arithmetic:

- Initial decision parameter at $(x_i, y_i) = (0, R)$:
  - $p_i = f(M_P) = f(x_i+1, y_i-1/2) = (x_i+1)^2 + (y_i-1/2)^2 - R^2$
    $= f(P) + 2x_i - y_i + 5/4 = 2x_i - y_i + 5/4 = 5/4 - R \cong 1 - R$



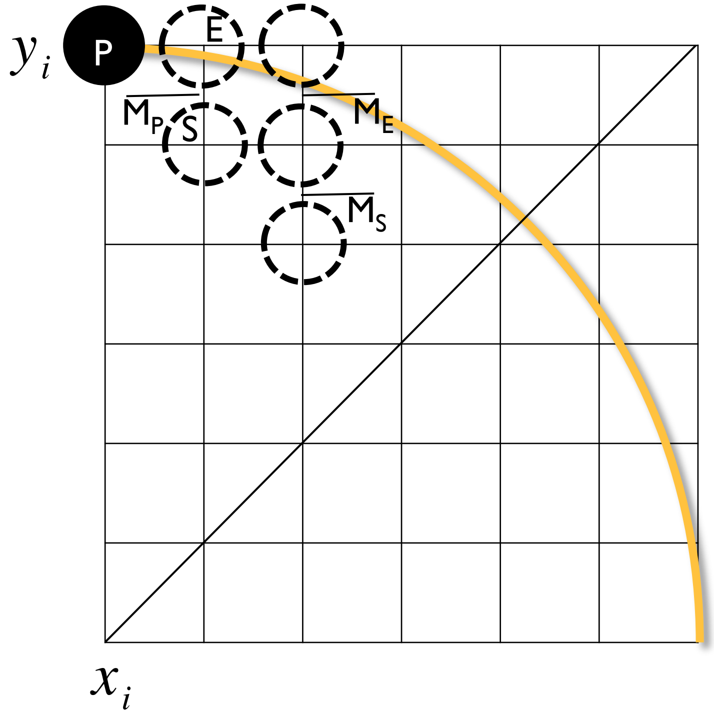
Current pixel:
E (East) or
S (South-East)

Midpoint circle algorithm (cont'd)

Algorithm:

```
void MidPointCircle(int R) {
        int x=0, y=R, d=1-R;

        DrawPixel(x,y);
        while (y>x)
        {
                if (p< 0)                // select E
                        p=p + 2 * x + 3;
                else                     // select S
                {
                        p = p + 2 * (x - y) + 5;
                        y = y - 1;
                }
                x = x + 1;
                DrawPixel(x,y);
        }
}
```



The algorithm only calculates the pixels on the 2nd octant. The remaining pixels are found using 8-way-symmetry.

SCAN CONVERSION

of

TRIANGLES/POLYGONS

Scan converting of polygons

Multiple tasks for scan conversion:

- Filling polygon (inside/outside)
- Pixel shading (color interpolation)
- Blending (accumulation, not just writing)
- Depth values (z-buffer hidden-surface removal)
- Texture coordinate interpolation (texture mapping)

Hardware efficiency critical

Many algorithms for filling (inside/outside)

Much fewer that handle all tasks well

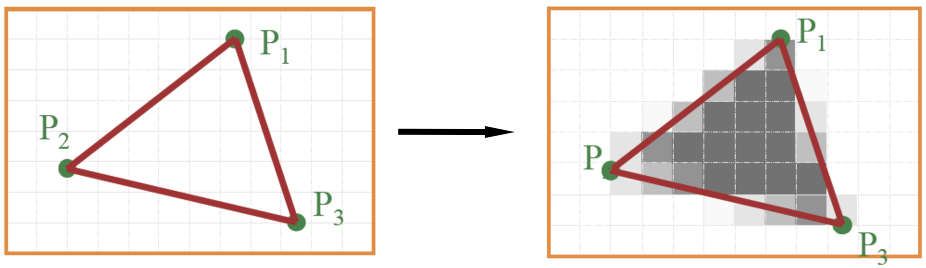
Review

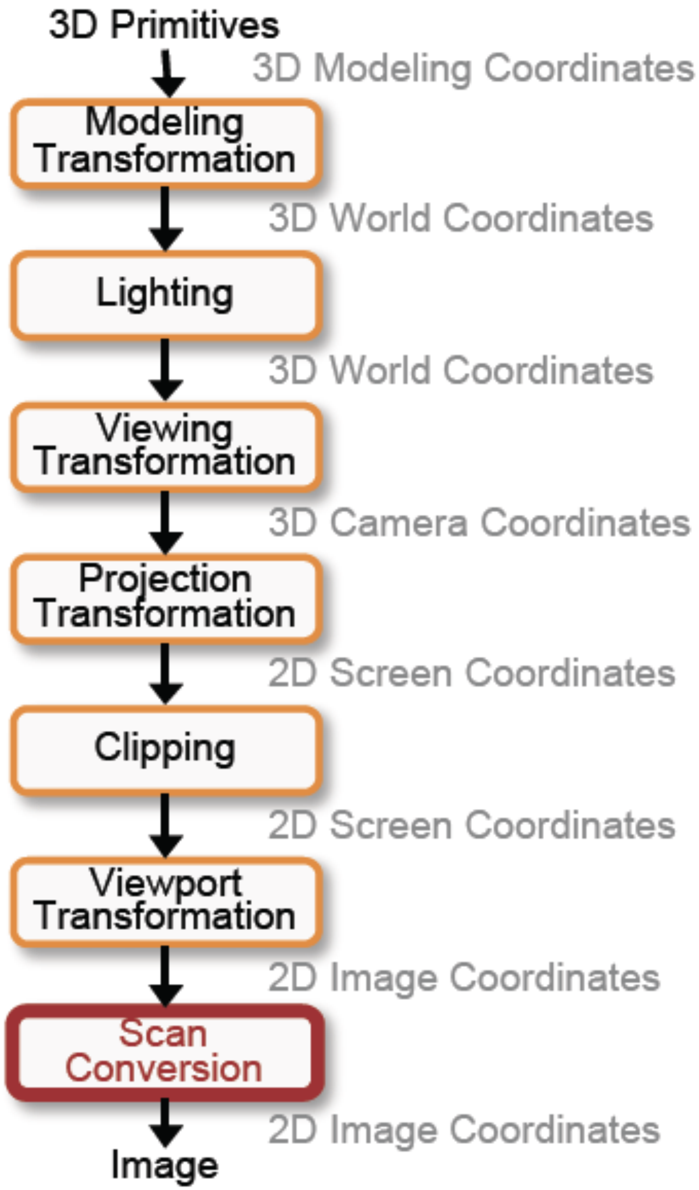
Shading:

- Determine a color for each filled pixel.

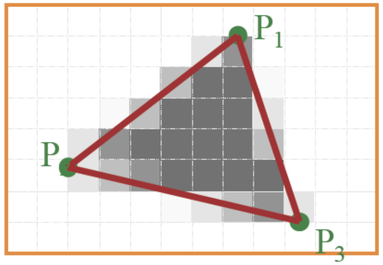
Scan conversion:

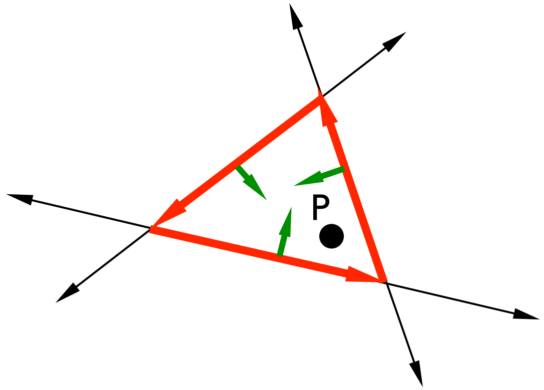
- Figuring out which pixels to turn on.
- Rendering an image of a geometric primitive by setting pixel colors.
- Example:
  - Filling the inside of a triangle.





Triangle scan conversion





Key idea:

- Color all pixels ***inside*** triangle.

Inside triangle test:

- A point is inside a triangle if it is in the positive half-space of all three boundary lines.
  - Triangle vertices are ordered counter-clockwise.
  - Point must be on the left side of every boundary line.
- Recall that the implicit equation of a line:
  - On the line: $Ax+By+C=0$
  - On right: $Ax+By+C<0$
  - On left: $Ax+By+C>0$

```
void ScanCTriangle(Triangle T, Color rgba)
{
    for each pixel P(x,y)
      if inside(P,T)
            setPixel(x,y,rgba)
}
```

```
Boolean inside (Triangle T, Point P)
{
    for each boundary line L of  T {
      float dot = L.A*P.x+L.B*P.y+L.C*P.z;
            if dot<0.0 return FALSE;
    }
    return TRUE;
}
```

Summary:

....

- Raster display technology.
- Basic concepts: pixel, resolution, aspect ratio, dynamic range, image domain, object domain.
- Rasterization and direct illumination.
- Graphics primitives and OpenGL.
- Geometry representations: explicit, parametric and implicit forms.
- Rasterization algorithms for straight line segments, circles and ellipses.
- Rasterization algorithms for triangles and polygons.
- Rasterization versus shading.